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# Inequality in Per Capita Water Availability: A Theil's Second Measure Approach

Desalination and Water Treatment

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## Inequality in Per Capita Water Availability: A Theil's Second Measure Approach

### **Abstract**

This paper puts forward the application of Theil's second measure in order to investigate international per capita water availability disparities. This index permits disparities to be disintegrated within and between groups of countries in a reliable way. An analysis of 188 countries for the period 1990–2012 demonstrates three observations: first, decline in per capita water availability disparities is characterized by both within-group and between-group inequality elements; second, between-group inequalities are at present the key contributor of the entire inequality scenario; and lastly, a comprehensive investigation on within-group inequalities divulges the noteworthy explanatory role played by Middle East countries towards increase in inequalities and countries pertaining to North America, Asia and Oceania towards decline in inequalities.

**Keywords:** Water availability; Theil index, Inequality; Decomposition analysis

## **1. Introduction**

Considering the recent literature on development economics, it can be seen that in due course of the inequitable and unbalanced economic growth across the world, inequality among the availability of basic needs is turning out to be predominant across the countries. If the individual countries are also being scrutinized, then it can be seen that the divide subsists between the rural and urban populations as well. This issue has been addressed by several researchers in diverse contexts [1, 2]. Considering the basic needs of life, water plays the most important role, as it is one of the primary elements of life. Apart from that, water is one of the major drivers in agricultural and industrial production, and therefore, in order to achieve a sustainable economic growth and to ensure food security, it is required for the nations to have sufficient availability of water resource.

However, by looking at the growth pattern of most of the nations across the world, it can be experienced that the world is going to encounter “Water Crisis” at a severe level. Keeping agriculture apart, constant increase in the demand of water for the purpose of non-agricultural consumption, which is majorly attributed to industrial usage, has placed demand of irrigation water under larger inspection, and this phenomenon has been appearing as water scarcity issue in various parts of the world [3]. Overgrowing demand of irrigated water has resulted in the transformations in flow of water, as the water is being diverted towards industrial production, and as a consequence, the qualities of stream water and ground water have worsened to a great extent [4–7], as well as its availability. Exploration of new water resources is also proving out to be economically and ecologically expensive enough, and as a result, the expansion of water base is not getting realized, thereby, restraining the water supply [8]. In addition to this, the issues related to climatic shift can worsen the situation [9].

In comparison with the figures of 1950s, demand for water across the globe has increased by more than three times, and alongside with that, the supply has been shrinking rapidly [10]. With the rise in global population, demand for irrigated water is expected to ascend in order to commensurate the requisites of food production, domestic, and industrial demand, and at the same time, inadequately and effortlessly obtainable freshwater resources in rivers, ponds, and groundwater aquifers are gradually diminishing owing to excess scale of utilization and water quality deterioration [11]. As agricultural sector is the most predominant user of water, this sector will be worst hit by water scarcity issues, resulting in endangering of food security [12, 13]. The issue regarding water scarcity can be intensified by the rate of growing expenses for exploring new water resources [14], dilapidation of land in irrigated areas [15], exhaustion of groundwater resources [16], and rapid formation of fecal coliform [11].

Now, looking at the countries across the world, it can easily be assumed that owing to their divergent growth patterns, the per capita availability of water varies to a great extent among them. Performance of countries regarding evaluation of their achievements in this regard can be carried out by means of well-defined indicators, so that the possible reallocation of resources can be suggested at all levels, may it be political or academic. Researchers have tried to formulate several indicators for capturing this aspect of inequality by linking it with several other economic aspects, and one of the predominant indicators in this case is Water Poverty Index (WPI), which has been designed in several ways in order to capture several economic aspects associated with availability of water [17–21]. However, in most of the cases, it has been seen that this indicator can prove out to be inconclusive in nature, as choice of the economic aspects can lead to subjectivity in terms of the performance of WPI, and this issue has been cited by several researchers, as well [22–26]. Moreover, calculations of WPI in most of the cases have been

carried out based on particular points of time, and the samples of those studies possibly fail to depict a composite global picture regarding the water poverty situation and distribution of per capita availability of water. Therefore, it is required to demonstrate the present global scenario regarding the inequality in per capita availability of water in a larger scale, using time series data and new indicator.

In this paper, an attempt has been made to compute inequality in per capita availability of water among 188 countries spanning over six groups<sup>2</sup> by means of Theil's second measure [27], as this index allows the divergences to be allocated within and between groups of countries in a reliable way. Historically, this index has been used to calculate inequalities in several contexts [28–34], and owing to its comparable nature, it is widely accepted among the researchers across the world. While analyzing the comparative inequality scenario, this index can provide with inter-temporal comparisons within individual groups, and across the groups, as well. Through the analysis carried out in this paper, it has been attempted to put forth both of the aforementioned aspects of comparative analysis, which have largely been ignored in the existing body of literature, considering the analysis of inequality in per capita availability of water or water poverty. Using Theil's second measure, this analysis is targeted towards addressing the research gap identified in the literature.

## **2. Material and methods**

The specialty of the Theil's second measure (see Appendix) is that it can be subdivided into two basic and comparable elements, namely a within-group inequality element, calculated as the weighted average of the intra-group inequality indices; and a between-group inequality element, denoting the inequalities that possibly can come into sight if only divergences subsist

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<sup>2</sup> Africa, Asia and Oceania, Central and South America, Europe, Middle East, North America

among averages of the individual groups [27, 35]. The index can be defined in the following manner:

$$T_i = \sum_{i=1}^n p_i \text{Log} \left( \frac{\bar{w}}{w_i} \right) \quad (1)$$

where,  $p_i$  stands for population percentage of country  $i$ ,  $w_i$  stands for per capita availability of water in country  $i$ , and  $\bar{w}$  stands for average per capita availability of water. In keeping with the standard mean logarithmic deviation and the approximations mentioned by Theil [27], range of Theil's second measure can be defined as (0, 1), where values approximated to zero can be considered as near to perfect equality condition, and values approximated to one as near to perfect inequality condition. The disintegration of  $T_i$  can be shown in the following manner:

$$T_i = T_{wg} + T_{bg} = \sum_{i=1}^g p_g \text{Log} \left( \frac{\bar{w}}{w_i} \right) \quad (2)$$

where,  $T_{wg}$  stands for the absolute within-group inequality element,  $T_{bg}$  stands for the absolute between-group inequality element, and  $p_g$  stands for population percentage of group  $g$ .

The annual data for per capita availability of water and population have been collected for the period of 1990–2012, and the sample includes 52 countries from Africa, 40 countries from Asia and Oceania, 36 countries from Central and South America, 43 countries from Europe, 13 countries from Middle East, and 4 countries from North America. Data for this study has been collected from World Bank indicators ([www.data.worldbank.org/indicator](http://www.data.worldbank.org/indicator)).

For the purpose of analysis, the entire dataset has been segregated into two parts, namely rural and urban, and this segregation has been done in order to demonstrate the inequality scenario not only between the groups, but also between the populace of the groups, who are divided by their level of income. First, the analysis will be carried out based on the aggregate data, followed by the segregated dataset, so that the comparative inequality scenario can be

demonstrated in an effective manner. Further analysis has also been carried out for individual groups, by segregating the dataset into rural and urban segments.

### **3. Results and discussion**

#### *3.1. Analysis of aggregate data*

The results for the aggregate data are shown in Table 1 and 2. The results recorded in Table 1 demonstrate the decomposition of Theil index considering all of the six groups, and the results recorded in Table 2 demonstrate the contribution of all of the six groups to within-group element. Fig. 1 demonstrates the graphical representation of the Theil indices, which are being calculated and recorded in Table 1. Now, we will look into these figures, so that some insights regarding the inequality scenario can emerge out.

*<Insert Table 1 here>*

*<Insert Table 2 here>*

*<Insert Fig. 1 here>*

In accordance with the obtained results, some observations have emerged, and those are as per the following:

First, the inequalities of per capita availability of water across the groups are showing a downward movement throughout the study period, and the Theil index has declined by nearly 76.77 per cent.

Second, the decomposition of Theil index into within-group and between-group elements can demonstrate the reason behind its sharp decline, which has hardly been looked into the literature, and was largely ignored. While assessing this result, it should be kept in mind that the gradual movement of the index towards equality has taken place in the context of rising access to improved water source. Therefore, keeping Middle East countries apart, rest five of the groups

have shown the downward trend, and Middle East countries have shown an upward trend throughout the period of study. This phenomenon can be attributed to the countries, like Iraq and Yemen, where the per capita availability of water through improved sources is radically poor, with populations more than the group average, thereby, adding to the inequality scenario.

Third, by looking at the obtained results, it can be easily seen that out of the two basic elements of the Theil index, the between-group element ( $T_{bg}$ ) contributes more towards the explication of the inequality scenario, as it accounts for almost 64.88 percent of the aggregate inequality. This element can prove out to be significant considering differential growth aspects among these nations, the distribution of population, the socio-economic structure of the nations, the technological advancements and ecological concerns, and lastly, geographical and climatic nature of the nations. Perhaps that is the reason behind the demonstration of low Theil index for the countries pertaining to Europe, North America, and Central and South America, and comparatively higher Theil index for the countries pertaining to Africa, Middle East, and Asia and Oceania. The inequality scenario for countries pertaining to Asia and Oceania has been improving radically over the years, whereas, for the African countries, even after the decline in Theil index, the inequality is quite higher compared to the other groups. Among all of the six groups, countries pertaining to Asia and Oceania have shown a decline of 94.02 percent in inequality, whereas North American countries have shown the same by 93.00 percent, followed by 88.41 percent for countries pertaining to Central and South America, 80.95 percent for European countries, and 44.25 per cent for African countries. Middle East countries have shown a rise of 54.31 percent in the inequality.

### *3.2. Analysis of rural population data*

*<Insert Table 3 here>*

*<Insert Table 4 here>*

*<Insert Fig. 2 here>*

The results for the data on rural population are shown in Table 3 and 4. The results recorded in Table 3 demonstrate the decomposition of Theil index considering all of the six groups, and the results recorded in Table 4 demonstrate the contribution of all of the six groups to within-group element, like the previous case. Fig. 2 demonstrates the graphical representation of the Theil indices, which is being calculated and recorded in Table 3. Now, we will look into these figures, so that some insights regarding the inequality scenario among the rural population can emerge out of the analysis.

In accordance with the obtained results, some observations have emerged, and those are as per the following:

First, the inequalities of per capita availability of water across the rural population of the groups are showing a downward movement throughout the study period, and the Theil index has declined by nearly 68.65 percent.

Second, the decomposition of Theil index into within-group and between-group elements can demonstrate the reason behind its sharp decline, as done in the previous case. While assessing this result, it should be kept in mind that the gradual movement of the index towards equality has taken place in the context of rising access to improved water source among the rural population. Therefore, keeping Middle East countries apart, rest five of the groups have shown the downward trend, and Middle East countries have shown an upward trend throughout the period of study. This phenomenon can be attributed to the countries, like Jordan and Yemen, where the per capita availability of water for rural population through improved source of water is radically poor and have shown a gradual decline throughout the period of study, with

populations more than the group average and amounting to nearly 41.60 percent, thereby adding to the inequality scenario.

Third, just like the previous case, by looking at the obtained results, it can be easily seen that out of the two basic elements of the Theil index, the between-group element ( $T_{bg}$ ) contributes more towards the explication of the inequality scenario, as it accounts for almost 52.57 percent of the aggregate inequality. This element can prove out to be significant considering differential rural development policies of these nations, their socio-economic and political balance, income distribution, geographical structure, agricultural land usage pattern, irrigation facilities, and infrastructural effectiveness. Perhaps that is the reason behind the demonstration of low Theil index for the countries pertaining to Europe, North America, and Central and South America, and comparatively higher Theil index for the countries pertaining to Africa, Middle East, and Asia and Oceania. The inequality scenario for countries pertaining to Asia and Oceania has been improving radically over the years, whereas, for the African countries, even after the decline in Theil index, the inequality is quite higher compared to the other groups. Among all of the six groups, countries pertaining to Asia and Oceania have shown a decline of 88.00 percent in inequality, whereas countries pertaining to Central and South America have shown the same by 87.11 percent, followed by 81.51 percent for European countries, 56.49 percent for North American countries, and 47.95 percent for African countries. Middle East countries have shown a rise of 104.67 percent in the inequality, which can be considered as an alarming situation in comparison to the other group of countries.

### *3.3. Analysis of urban population data*

The results for the data on urban population are shown in Table 5 and 6. The results recorded in Table 5 demonstrate the decomposition of Theil index considering all of the six

groups, and the results recorded in Table 6 demonstrate the contribution of all of the six groups to within-group element, like the previous case. Fig. 3 demonstrates the graphical representation of the Theil indices, which is being calculated and recorded in Table 5. Now, we will look into these figures, so that some insights regarding the inequality scenario among the urban population can emerge out of the analysis.

*<Insert Table 5 here>*

*<Insert Table 6 here>*

*<Insert Fig. 3 here>*

In accordance with the obtained results, some observations have emerged, and those are as per the following:

First, the inequalities of per capita availability of water across the urban population of the groups are showing a downward movement throughout the study period, and the Theil index has declined by nearly 14.60 percent, apart from small rises of 11.65 percent in 1994 and 8 percent in 2008.

Second, the decomposition of Theil index into within-group and between-group elements can demonstrate the reason behind its sharp decline, as it was seen in the previous two cases. While assessing this result, it should be kept in mind that the gradual movement of the index towards equality has taken place in the context of rising access to improved water source among the urban population. In comparison with the previous two cases, this case is more critical considering the inequality conditions, as four out of six groups of countries are showing rise in inequality. Countries pertaining to Asia and Oceania and North American have shown downward trends in inequality, whereas, Africa, Central and South America, Europe and Middle East have shown upward trends in inequality.

Third, just like the previous two cases, by looking at the obtained results, it can be easily seen that out of the two basic elements of the Theil index, the between-group element ( $T_{bg}$ ) contributes more towards the explication of the inequality scenario, as it accounts for almost 62.76 percent of the aggregate inequality. This element can prove out to be significant considering differential urban development policies of these nations, their socio-economic and political outlook towards environmental protection, income distribution, inhabitation structure, industrial land usage pattern, and infrastructural effectiveness. Perhaps that is the reason behind the demonstration of low Theil index for the countries pertaining to North America, and Asia and Oceania, and comparatively higher Theil index for the countries pertaining to Africa, Middle East, Europe, and Central and North America. The inequality scenario for countries pertaining to Asia and Oceania has been demonstrating gradual improvement over the years, whereas, for the African countries, the inequality has been increasing radically. The inequality scenario in countries pertaining to Europe and Central and South America has been fairly consistent throughout the study period. For the North American countries, the inequality dropped radically in the year 1992, and since then it has shown a slow but gradual rise. After showing a steep rise, Middle East countries have shown a radical decline in the inequality in the year 2004, and since then they have also shown a slow and gradual rise. Among all of the six groups, countries pertaining to North America have shown a decline of 86.89 percent in inequality, followed by 56.61 percent decline in countries pertaining to Asia and Oceania. On the other hand, countries pertaining to Africa have shown a rise of 419.66 percent in inequality, which can be considered as extremely alarming. They are followed by 86.63 percent rise in inequality for Middle East countries, 66.68 percent rise for European countries, and 4.63 percent rise for Central and South American countries.

In a nutshell, except Middle East countries, remaining five of the six groups have shown downward trends in the inequality scenario, while considering the aggregate population data, and this explanation changes in accordance with the segregation of the population data into rural and urban. Except for Middle East countries, remaining five of the six groups have shown downward trends in the inequality scenario, while considering the rural population data, and except for the countries from Asia and Oceania and North America, remaining four of the six groups have shown upward trends in the inequality scenario, while considering the urban population data.

#### **4. Conclusion**

By far, using Theil's second measure, the inequality in per capita availability of water from improved sources among the countries from the six major groups of the world has been analyzed for the period of 1990–2012, and it has been seen that leaving the particular cases apart, the inequality is coming down. However, the problem of inequality can prove to be severe for the urban population of most of the countries in comparison with the rural population, and amongst all of the six groups, Middle East countries have demonstrated rise in inequality in both the cases, and countries pertaining to North America, Asia and Oceania have demonstrated decline in inequality in both the cases.

Apart from this, the formulation of this index has revealed two major points, namely, the changes in the inequality scenario can be demonstrated by bifurcation of the index into within-group and between-group elements, and out of these two, between-group element contributes more significantly to the inequality. Briefly, these results can bring out two harmonizing themes of research regarding water strategies; first, the practices, through which a convergence towards per capita availability of water can be achieved in the context of increasing availability of water from improved sources, can be examined; and second, elucidation of the means those add to the

intercontinental dispersion of technological mechanisms, international relations, environmental concerns, and utilization pattern, and thereby, leading towards divergences in the per capita availability of water among the different classes population of the nations.

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## Appendix

In keeping with the information entropy measure [36], Theil's index can be derived, and the universal form of entropy is given by the following:

$$E = -k \sum_1^N (p_i \log p_i) \quad (3)$$

where,  $p_i$  is the probability of finding income  $y_i$  of a person among the population of  $N$ , and the total income of the population can be given by  $N\hat{y}$ ,  $\hat{y}$  being the average income of the population.

Therefore, the observed entropy represented by Theil's index is given by:

$$E = \sum_1^N \left( \frac{y_i}{N\hat{y}} \log \frac{N\hat{y}}{y_i} \right) \quad (4)$$

Assuming the homogeneity among the population, it can be stated that  $p_i = 1/N$ . In that case, Eq. 4 takes the following form:

$$E = \frac{1}{N} \sum_1^N \left( \log \frac{N\hat{y}}{y_i} \right) \quad (5)$$

It is the limiting condition imposed on Theil's basic measure, where the scalar multiplier value is approximated to zero [35], as per the following:

$$E = \lim_{c \rightarrow 0} \left[ \frac{1}{N} \frac{1}{c} \sum_1^N \left\{ \left( \frac{y_i}{N\hat{y}} \right)^c - 1 \right\} \right] = \frac{1}{N} \sum_1^N \log \left( \frac{N\hat{y}}{y_i} \right) \quad (6)$$

This is the form of Atkinson's index [37] along the lines of a utilitarian social welfare function with utility of income presented in a logarithmic form. This form is commonly known as Theil's second measure.