Investment and wage gap in India: a general equilibrium analysis

Subhasankar Chattopadhyay
Department of Economics, Indian Institute of Management Indore, Indore, India

Received 11 December 2017
Revised 9 January 2018
23 March 2018
30 March 2018
13 July 2018
20 July 2018
Accepted 23 July 2018

Abstract

Purpose – This paper aims to theoretically find out whether investments could close the formal-informal wage gap in India.

Design/methodology/approach – The paper builds a general equilibrium model of a developing economy with a large informal sector and a capital-intensive formal sector with sector-specific capital and incorporates endogenous demand.

Findings – With homothetic preferences, a small initial wage premium and elastic relative demand, investment in the formal sector is likely to close the wage gap, but the gap persists with non-homothetic preferences. However, investment in the informal sector is unlikely to close the wage gap with either type of preferences.

Originality/value – Though labour market distortions in developing economies leading to a formal-informal wage gap are well-documented in the development literature, little attention has been given to the question of whether such a gap would close over time.

Keywords Investment, Wage gap, Formal and informal sectors, General equilibrium

Paper type Research paper

1. Introduction

Labour market distortions leading to a formal-informal wage gap in developing economies is a well-known fact in the development literature (Agenor and Montiel, 2008; Bargain and Kwenda, 2009; Daza and Gamboa, 2013; Karan and Selvaraj, 2008; Kumar and Ranjan, 2015; Marjit and Kar, 2009, 2011; Narayanan, 2015; Tansel and Kan, 2012). However, not enough attention has been given to the question of whether such a gap would close over time. Such a question is not a new one – it is similar to the well-known convergence debate (Barro and Sala-I-Martin, 2003) that has been deeply explored in the neo-classical model. Here we have two sectors, formal and informal – instead of countries – for which the possibility of wage equalisation arises.

With convergence in mind, it is not too hard to identify the possible candidate that might drive the wages – investments across sectors[1]. While investments in the formal sector is well-recognised, the informal sector (which also includes agriculture) in developing economies is nothing short of a thriving' sector; the size of the informal sector across countries by alternative measures is huge (La Porta and Shleifer, 2014). Both the informal manufacturing units and self-employed units accumulate fixed assets, invest and grow (Chattopadhyay and Mondal, 2017).

What has been the trend in the wage gap in India? Figure 1 presents the trend in the (average) formal[2] and informal annual real wages (in the manufacturing sector) over the past four NSSO (National Sample Survey Organization, India)[3] rounds and movements in the wage gap. The wage gap is not constant and seems to have reduced between 1994-95 and 2005-06, but has increased thereafter. The reduction in the first three rounds was primarily due to rises in the informal wage rates[4].

Indian Growth and Development Review
Vol. 11 No. 2, 2018
pp. 107-135

1753-8254
DOI 10.1108/IGDR-12-2017-0104

The calculation of the wage gap requires further attention. It may be that the average wage gap does not emerge at all the skill levels of the workers across sectors – one needs comparable skill levels to see whether the wage gap is indeed converging. Such an exercise is often difficult as the informal sector is likely to have a high degree of heterogeneity, and the data rarely account for the unobserved individual characteristics of workers (Bargain and Kwenda, 2009). However, using large (unbalanced) panels and fixed effects quantile regression for data sets of South Africa, Brazil and Mexico, Bargain and Kwenda (2009) found that all the three countries show a similar pattern once workers' heterogeneity is accounted for – informal sector wage gaps are significant in the lower part of the distribution but tend to disappear at the top.

Narayanan (2015) also uses a quantile regression technique to test for labour market segmentation in the Indian labour market by using the NSSO data on Employment and Unemployment for the 2011-12 round. For men, while the coefficient effects[5] explain the major part of the wage gap between the 10th and the 40th quantiles, the endowment effect explains a major part of the wage gap at higher quantiles. For women, the endowment effect explains the major part of the wage gap across the whole distribution. By both these studies, therefore, there is evidence that the wage gap exists even when the skill level is controlled for. However, as Narayanan's study involves a single round, it cannot be ascertained what has been happening to the wage gap over time[6].

The existing literature has addressed the issue of a wage gap in several ways. One of the ways to distinguish between the formal and informal sector is to model the informal sector as a shadow economy (Frey and Schneider, 2000; La Porta and Shleifer, 2008, 2014; Schneider, 2012; Schneider *et al.*, 2003). A shadow economy constitutes un-recorded yet productive activities which should otherwise be a part of the national product. As the informal sector – viewed in this way – is something that is beyond a legal framework, it often flouts minimum wage laws and pays lower wages than the formal sector, giving rise to a wage gap. Next, there is a large body of literature that attributes differences in the skills of workers to the formal-informal wage gap, with a presumption that the formal sector employs skilled labour – see Marjit and Acharya (2003), Marjit and Kar (2011), and Santra (2014) and the references therein for a detailed survey of this literature. This literature aims to explain why the skilled-unskilled wage gap has increased over the past few decades.

Another side of the literature models the formal sector as an unionised sector, with the bargaining outcome being a wage premium in the formal sector (Chaudhuri, 2003, 2008; Chaudhuri and Banerjee, 2007, 2010; Chaudhuri and Yabuuchi, 2007; Farber 1986; Marjit, 2003, Marjit and Kar, 2009, 2011; Yabuuchi and Chaudhuri, 2007). Such a wage premium

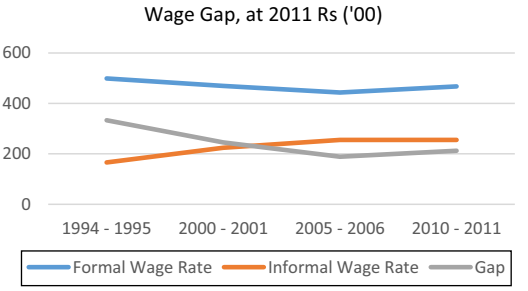

Figure 1.
India – formal and informal (average) real wage levels and gap

Source: Various rounds of NSSO, India

leads to what is known as labour market distortion (Jones, 1971; Neary, 1978) as the unionised sector gets a wage that is over and above the otherwise market clearing wage. However, these models primarily focus on short-run comparative statics effects of some policy intervention, but do not trace the wage gap *over time*. For instance, Chaudhuri and Banerjee (2007) have analysed the consequences of economic reforms on the well-being of the informal sector workforce using a three-sector general equilibrium model and have found that different liberalised policies might produce diverse effects on the informal wage, a result that is independent of the condition of inter-sectoral capital mobility. Chaudhuri and Banerjee (2010) analyse a three-sector general equilibrium framework to investigate the possibility of unemployment of both skilled and unskilled labour and find that the effects of inflows of foreign capital on skilled–unskilled wage inequality and the extent of unemployment of both types of labour depends on the efficiency of skilled workers. Gupta and Dutta (2011) examined the effects of changes in different factor endowments and prices of traded goods on the unemployment rates and on the skilled–unskilled relative wage, with perfect capital mobility. Marjit *et al.* (2007) have shown that better prospects for agricultural exports and productivity need not necessarily increase the agricultural wage – it depends on the extent of capital movement between the formal and informal manufacturing sectors. Acharyya (2011), in the context of trade, has examined the effects of conversion of one type of physical trade restrictions into another on intra-country wage inequality in a standard Heckscher–Ohlin–Samuelson model. However, all such studies use *static* competitive general equilibrium modelling.

Though Lingens (2007) analysed the *growth* effects of union wage bargaining in a growth model with expanding product variety, the question of a wage gap was not taken up. Chattopadhyay and Mondal (2017) have done a growth theoretic general equilibrium analysis but have stayed away from endogenous demand. García-Peñalosa and Turnovsky (2005) and Turnovsky and Basher (2009) used an intertemporal growth theoretic framework but did not capture labour market distortions. Hence, available general equilibrium models do not directly deal with the question of the wage gap *over time*.

Motivated by the absence of studies that focus on a growth theoretic framework with factor market distortions, this paper presents a general equilibrium model of a developing economy with a large informal sector, a capital-intensive formal sector with sector-specific capital and endogenous demand to uncover the possible roles of investments and factor intensities in determining the trend in the formal-informal wage gap over time. The analyses show that closing of the wage gap depends on three factors – preference type, capital intensity condition and the extent of the labour market distortion. Investment in the formal sector is likely to close the gap when the formal sector is capital intensive, preferences are homothetic and the labour market distortion is not deep (i.e. when initial wage premium is small). However, investment in the informal sector is unlikely to close the gap with either type of preferences.

2. The model

Our model is a variant of the general equilibrium models used by Marjit and Kar (2009, 2011), Chaudhuri (2003, 2008) and Chaudhuri and Banerjee (2007, 2010)[7]. We have a two-sector closed economy with a formal (Y-sector) and an informal sector (X-sector). Output and input markets are competitive, and the sectors use constant returns to scale (CRS) production technologies. Both sectors use capital and labour, labour is perfectly mobile, but capital is sector-specific. So capital stocks do not have alternative use either in the short run or in the long run[8]. In the formal sector, nominal wages are negotiated by bargaining – the formal sector is unionised. The bargained wage is more than the otherwise market clearing

wage, hence, the formal workers earn a wage premium over their informal counterparts. This premium is the source of labour market distortion (Agenor, 1996; Agenor and Montiel, 2008; Carruth and Oswald, 1981; Chattopadhyay and Mondal, 2017; Chaudhuri, 2003, 2008; Chaudhuri and Banerjee, 2010; Chaudhuri and Yabuuchi, 2007; Cole and Sanders, 1985; Fields, 1990; Jones, 1971; Marjit, 2003; Marjit and Acharya, 2003; Marjit and Kar, 2009, 2011; Kar and Marjit, 2001; Marjit and Beladi, 2002; Mazumdar, 1983, 1993; Neary, 1978; Turnham, 1993; Yabuuchi and Chaudhuri, 2007). However, such a wage gap does not lead to open unemployment in the Harris-Todaro sense in our model. With a given the level of the formal capital stock and fixed bargained wage in the short run, marginal productivity determines the formal-labour requirement. The residual (unemployed) labours are poor and therefore cannot afford to wait to find jobs because of survival needs and eventually get absorbed into the informal sector (Basu, 2013; Marjit and Kar, 2009, 2011). Given the level of informal capital stock, the level of residual labour determines the informal wage rate. These conditions ensure full employment of labour.

Our approach to modelling is different from existing studies in at least two important ways. First, we model the formal sector as a unionised sector rather than modelling the informal sector as a shadow economy. Typically, a shadow economy, as by definition constitutes unrecorded activities, can potentially include formal and/or informal firms. Further, it is not possible to get reliable survey data on the shadow economy on the basis of which a meaningful analysis of wage gaps can be done[9]. Therefore we follow the standard general equilibrium modelling with full employment where employment in the informal sector is determined residually as discussed above. Second, available general equilibrium models that study labour market distortions are at best *static* general equilibrium models and do not directly deal with our question of the wage gap *over time*. We allow capital accumulation, rather than studying comparative statics effects of a one-shot change in the level of capital stock, induced by rates of return differential across formal and informal sectors. Further, we also incorporate endogenous demand conditions of both the homothetic and non-homothetic variety so that a complete characterisation can be obtained. Such comprehensive treatment is expected to enrich the existing varieties of models.

Following the standard assumption of the related literature, we assume that the formal sector is capital-intensive and the informal sector is labour intensive (both in terms of physical and value sense). This is going to be our capital intensity hypothesis (CIH) throughout. Let all durable goods be produced in the formal sector and let all non-durables (which includes mass consumption goods as well) be produced in the informal sector. From the demand side, (Section 2.3), however, final outputs are substitutes.

With sector-specific capital, it is straightforward that the rates of return to the sectoral capital are not equalised in the short run. Therefore, the economy begins with capital market disequilibrium (unequal capital rates) along with a labour market distortion (formal-informal wage gap) – call this the short run. Our objective is to find out the effects of investments going either to the formal or to the informal sector on the trend of the formal-informal wage gap till capital rates are equalised (i.e. till the capital market equilibrium is attained) – call this the long run[10]. The investment goods sector is exogenous to the model, including the possibility of that it lies outside the country (say of foreign direct investment variety).

The equilibrium of the model consists of three sets of equations:

(1) price equations from zero profit conditions;

(2) commodity market equilibrium; and

(3) full employment or factor market equilibrium conditions. Note that in both the sectors, firms are maximising their profit as marginal productivity conditions are satisfied. Markets are competitive and therefore profits in Y and X sectors are zero. The following notations are used.

Y, X: outputs in the formal and informal sectors.

K_Y, K_X: capital stocks in the formal and informal sectors.

K, L: total supply of capital and labour in the short run (with fixed labour endowment throughout).

w_Y, w_X: real wage rates (in units of X-goods) in the formal and informal sectors.

r_Y, r_X: return to capital (in units of X-goods) in the formal and informal sectors a_{Ki} and a_{Li} are the respective capital and labour requirements to produce one unit of output (inverse of average productivity of the factor) in the sectors 'Y' and 'X'.

2.1 Price equations

Under zero-profit conditions (as markets are perfectly competitive), the unit output price is paid to either capital or labour:

$$p = r_Y a_{KY} + w_Y a_{LY} \tag{1}$$

$$1 = r_X a_{KX} + w_X a_{LX} \tag{2}$$

Here, p is the relative price of Y-goods in units of X-goods ($\equiv P_Y/P_X$), which can also be termed as inter-sectoral terms of trade. Rates of return (r) and wages (w) are in units of X-goods.

Note that $w_Y = W_Y/P_X$ and $w_X = W_X/P_X$, upper case variables are nominal values and corresponding lower-case variables are real values (in units of X). W_Y is the bargained nominal wage in the formal sector and is fixed to begin with. Note that, due to a wage premium, $w_Y \neq w_X$.

The conversion to relative prices with respect to non-durable goods (here X-goods) is done keeping in mind the homogeneity postulate[11] – that the demand and supply depend on relative prices and not on absolute prices. Furthermore, rates of return and wages must be compared in terms of a common numeraire. Here, we use non-durables as the numeraire.

2.2 Supply condition

At any given point in time, the available capital stock is sector-specific and is given by history. The total capital stock changes over time with investment. The supply conditions along with mobile labour under full employment are given by:

$$K_Y = a_{KY} Y \tag{3}$$

$$K_X = a_{KX} X \tag{4}$$

$$\overline{K} = K_Y + K_X \tag{5}$$

$$\overline{L} = a_{LY} Y + a_{LX} X \tag{6}$$

along with:

$$\left.\begin{array}{l} a_{Li} = a_{Li}(w_i/r_i); i = Y, X \text{ and } da_{Li}/d(w_i/r_i) < 0 \\ a_{Ki} = a_{Ki}(w_i/r_i); i = Y, X \text{ and } da_{Ki}/d(w_i/r_i) > 0 \end{array}\right\} \quad (7)$$

Though capital is-sector specific, equation (7) ensures full employment of both the factors at each point in time.

2.3 Demand condition

In multi-sector models, equilibrium involves movement of relative prices. Changes in relative prices influence both demand and supply, but in the very short period, output changes may not be possible. So the burden of adjustment falls on demand and thereby on relative prices. In aggregative models, the choice on the demand side is essentially between consumption and saving. While this can be modelled in terms of usual dynamic utility maximisation and individual choice, for developing economies, income distribution is equally important. Therefore, we are going to take up both varieties of demand functions i.e. preferences, namely, homothetic and non-homothetic.

Note that the supply functions in our models differ only in terms of capital intensity, but not in terms of productivity. Introduction of non-homothetic preference also generate demand-side imbalances that are extremely important to tracing the behaviour of an economy over time. For instance, non-homothetic preferences are now a standard device to investigate structural change. Such preferences are particularly manifested at the initial stages of development in a developing economy (Herrendorf *et al.*, 2014 for more detail). Non-homothetic preferences are a neat way to capture Engel's law, which states that a larger proportion of income is spent on income and price elastic goods (such as non-agricultural goods in general) and less is spent on income and price inelastic goods (necessities such as food) as households' income rise.

Assume that workers have only wage income (but no capital income). This means they do not spend on investment goods – they consume the entire wage income. This assumption is not unreasonable in the context of developing economies. One way to invest in capital is to indirectly hold financial claims – issued by firms – through financial intermediaries. As per the World Bank's Global Financial Inclusion Index (Global Findex, 2014), only 27 per cent of the world's population (over age 15) have "formal" savings. The figure stands at 13 per cent for South Asia, 14 per cent for Latin America and 14 per cent for India. In terms of income groups, 10 per cent of the lower and 15 per cent of lower-middle income groups have formal savings. In the year 2011, only 18.65 per cent of the poorest 40 per cent in India saved *some* money in the previous year. The number reads 20 per cent for rural India. Workers are most likely to belong to low- and low-middle income groups (in the formal sector) and to rural areas (a large part of the informal sector is the agricultural sector). For analytical tractability, we assume that their saving rate is zero[12].

The owners of capital – the so-called capitalist – save a fraction of their income and invest. Rest of the capitalists' income is spent on the formal and informal goods. Within this broad saving behaviour, we try two different specifications to model demand structure. In the first variety, we assume that the workers consume only the X-goods and the capitalists consume all the Y-goods, i.e. a fixed proportion of capitalists' income is spent on Y-goods and the rest on the investment goods (therefore relative demand between them is unit elastic). When X-goods market is in equilibrium, then by Walras law, Y-sector is also in equilibrium. The demand-supply balance for X-goods given by:

$$D_X = X = w_Y L_Y + w_X L_X \tag{8}$$

helps us determine the relative price at a point in time (Section 2.6). For brevity, let us call this preference as non-homothetic preference[14].

The other variety of demand structure comes from the usual homothetic utility function involving spending on both Y- and X-goods by workers and capitalists. Such utility functions makes the relative demand to be a function of relative price only (Jones, 1965).

$$\frac{Y_D}{X_D} = f(p); f' < 0; \tag{9}$$

Note that changes in income do not affect the position of the relative demand curve as expenditure shares in homothetic preference is independent of income at a given level of p – demand function is unitary income-elastic.

2.4 Wage setting rule

The wage setting process can be modelled either as a monopoly model of trade union or as a Nash bargaining solution (also known as the "right to manage" model). Relegating the explicit solution of the problem to Appendix 1, we present here the set-up of the monopoly model of the trade union, and provide a graphical analysis of the equilibrium outcome.

Consider the following variety of utility function for the labour union where the choice set of the representative union includes both the wage rate and the level of employment:

$$U(w_Y, L_Y) = L_Y[u(w_Y) - u(w_X)] \tag{10}$$

Here, w_X – the informal wage rate – is the fall-back or outside option to the formal workers. The union treats this outside option parametrically. In the monopoly model, the union first sets the wage and then the representative firm choses the level of employment so as to maximise its profit (Heijdra, 2009; Ch 7).

The union's objective is then to:

$$\underset{w_Y}{Max}\ U(w_Y, L_Y)$$

Subject to:

$$\frac{\partial}{\partial L_Y}\pi_Y(w_Y, L_Y; K_Y) = 0$$

This exercise determines w_Y and L_Y. The informal-firm maximises its profit which gives us the following two conditions, which in turn determine L_X and w_X:

$$\frac{\partial}{\partial L_X}\pi_X(w_X, L_X; K_X) = 0$$
$$L_Y + L_X = \overline{L}$$

Here, π_Y and π_X are profit functions of the representative formal and informal firm, respectively.

Using the first-order conditions, it is possible to express the equilibrium relation between the formal and informal wages in the following reduced form (Appendix 1):

$$w_Y = \alpha(w_X) \tag{11}$$

It is evident that the economy begins with a wage premium that leads to a formal-informal wage gap. This premium is over the otherwise market clearing wage[15]. The equilibrium outcome is shown in Figure 2.

An increase in w_X (say due to investment in the formal sector – see Figure 2) causes a less-than proportionate increase in w_Y, so that, in terms of elasticity, $0 < e_\alpha < 1$ for $w_Y > w_X$ and $e_\alpha = 0$ for $w_Y = w_X$. It is immediate from (11) that:

$$\hat{w}_Y = e_\alpha \hat{w}_X \tag{12}$$

Chaudhuri (2003, 2016), Chaudhuri and Banerjee (2007), Chaudhuri and Yabuuchi (2007), Chaudhuri and Biswas (2016) and Yabuuchi and Chaudhuri (2007) use a similar inelastic informal-to-formal wage adjustment rule.

The rule given by equation (12) is applicable till wages equalise. After equalisation, w_Y and w_X move with equality; hence, $\hat{w}_Y = \hat{w}_X$.

2.5 Determination of variables

Given an initial relative price of Y-goods in terms of X-goods (assuming both goods are produced) say p_0, the distribution of the labour force, the level of equilibrium wage rate in the X-goods sector and the income of capital K_Y and K_X can be determined with the help of marginal product of labour curves as shown in Figure 2. The length of the horizontal axis equals the total labour supply available at a point of time. Note that the position of MPL_X curve is independent of the relative price p.

Once w_Y is determined, the position of $pMPL_Y$ curve determines L_Y. The wage rate in X-sector, w_X can be determined from the requirement that the residual labour $L - L_Y = L_X$ must be fully employed in the informal sector. The income of labour and capital in Y- and X-sectors are given by the corresponding areas of rectangles under the respective wage line and by the triangular area above the equilibrium wage line, respectively.

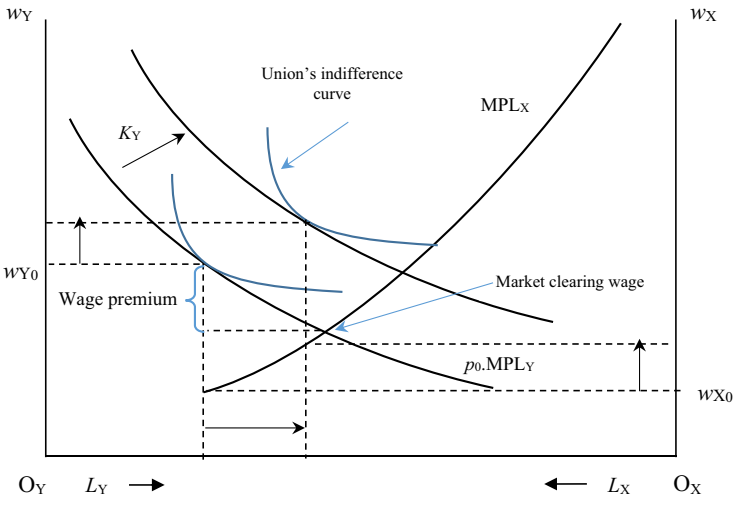

Figure 2.
Determination of wage rates, allocation of labour and equilibrium effects on wages with investment in the formal sector

It is immediate from Figure 2 that the wage rates and allocation of labour are functions of the structure of sectoral capital stocks, total supply of labour and commodity prices. Changes in any of these would change the equilibrium wage rate and allocation of labour, and hence outputs in both the sectors.

How is the short-run 'p' determined? That is answered by looking at the demand conditions.

2.6 Determination of relative price

That the relative demand curve for X-goods is downward sloping in case of homothetic preference is straightforward from equation (9).

Next consider the non-homothetic case. A fall in p (or a rise in $1/p$) – which is equivalent to a rise in P_X – causes $p.MPL_Y$ curve to shift left. This causes L_Y to decrease at a given w_Y, so that wage bill $w_Y L_Y$ in the Y-sector decreases. A fall in L_Y will cause L_X to rise and w_X to fall. If the MPL_X is relatively inelastic then $w_X L_X$ will fall (a given rise in L_X requires w_X to fall by a large amount). So the aggregate wage bill $w_Y L_Y + w_X L_X$ will fall. But there are further adjustments; a fall in w_X causes a secondary fall in w_Y by equation (11). So let us find out the conditions under which the relative demand curve is downward sloping. We need the aggregate wage bill (equalling demand D_X) to fall with a rise in $1/p$.

From equation (8), $D_X = w_X L_X + w_Y L_Y$ and totally differentiate both sides to get:

$$\hat{D}_X = \theta_X(\hat{w}_X + \hat{L}_X) + \theta_Y(\hat{w}_Y + \hat{L}_Y)$$

where θ_i is the ith sector's labour income share in total labour income and an over hat implies proportionate rate of change of a variable.

Using equations (6) and (12):

$$\hat{D}_X = \theta_X(\hat{w}_X + \hat{L}_X) + \theta_Y\left[e_\alpha \hat{w}_X - \frac{\lambda_{LX}}{\lambda_{LY}}\hat{L}_X\right]$$

$$\hat{D}_X = [\theta_X + \theta_Y e_\alpha]\hat{w}_X + \left[\theta_X - \theta_Y \frac{\lambda_{LX}}{\lambda_{LY}}\right]\hat{L}_X$$

where λ_{Li} is the fraction of labour employed in the ith sector.

Under full employment, $\hat{w}_X$and $\hat{L}_X$ must have opposite signs (which is also evident from Figure 2). A rise in $1/p$ causes L_Y to decrease and L_X to increase (therefore w_X to decrease), which in turn causes the total wage bill to decrease in the case when there are no further adjustments. We want this result to be preserved even when further adjustments through equation (12) is allowed. Therefore, after all adjustments due to a rise in $1/p$ are over, we should have $\hat{w}_X < 0$ and $\hat{L}_X > 0$[16]. What parametric restrictions are required for this to hold?

For $\hat{D}_X > 0$ we must have $[\theta_X + e_\alpha \theta_Y] > 0$ and $\left[\theta_X - \theta_Y \frac{\lambda_{LX}}{\lambda_{LY}}\right] < 0$.

As $0 < e_\alpha < 1$ and $0 < \theta_X, \theta_Y < 1$, we have $[\theta_X + e_\alpha \theta_Y] > 0$. Hence, the sufficient condition for demand to fall is given by $\left[\theta_X - \theta_Y \frac{\lambda_{LX}}{\lambda_{LY}}\right] \leq 0$.

$$i.e., \theta_Y \frac{\lambda_{LX}}{\lambda_{LY}} \geq \theta_X,$$

$$or, \frac{\lambda_{LX}}{\lambda_{LY}} \geq \frac{\theta_X}{\theta_Y}$$

$$or, \frac{L_X}{L_Y} \geq \frac{w_X L_X}{w_Y L_Y}$$

or, $w_Y \geq w_X$, which is the case by the structure of the model. Therefore, the relative demand curve for X-goods is downward sloping. Note that we do not need a CIH – either in physical or in the value sense – to be satisfied for the relative demand curve to be downward sloping. From the properties of the relative demand curve, the following proposition is immediate.

Proposition 1. With homothetic preference, the relative demand curve for formal goods is always downward sloping. With non-homothetic preference, the relative demand curve for formal goods is downward sloping provided the formal-informal wage gap is strictly non-negative. Both properties do not require a CIH.

Now about the relative supply curve. A rise in L_X raises X given K_X. Thus, a rise in $1/p$ raises X – the standard short-run upward rising supply curve for X (Figure 3). Note two points; first, the supply and demand curves are drawn for given K_Y and K_X. A change in either of them will cause both demand and supply curves to shift. Second, the determination of p tells us how the level of the equilibrium price is determined in the short-run, and this is not the long-run p.

2.7 Model closure

The model has eight unknowns – w_X, p, r_X, r_Y, Y, X, L_X and L_Y. We have eight equations, given by two price equations (1) and (2), X-goods market equilibrium [equation (8)], full employment of two sector-specific capital [equations (3) and (4)], full employment of labour [equation (5))] and that the mobile factor (here labour) receives it marginal product in each sector (two equations).

The condition $D_X = X$ determines p. From p, one can determine w_Y by referring to the labour allocation diagram (Figure 2). Once w_Y and p are known [w_X is obtained from equation (11)], r_Y and r_X get determined from equations (1) and (2). Also, w_Y determines L_Y and the condition that there is no open unemployment of labour determines L_X. The labour allocation along with given K_Y and K_X determine Y and X through the production functions. Additional four input coefficients a_{ij} are determined by the set of four equations given by equation (7) by specifying w_Y, w_X, r_Y and r_X.

3. Investment and growth

Much of the related literature cited in Section 2 is about reallocation of the *existing* capital stock across sectors over time. Such an adjustment mechanism is essentially Marshallian in

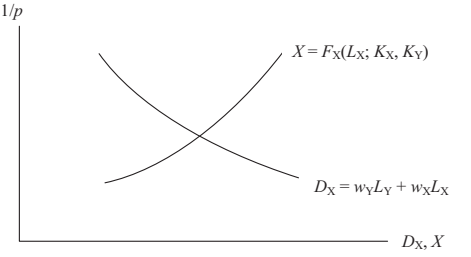

Figure 3.
Determination of p: relative demand and supply curves for x-goods

nature, where long-run is the time by which all reallocations, with a fixed *aggregate* capital stock, are complete. However, that is not about a change in the total available stock of capital – it is just about *reallocating* a *given* level of the capital stock. In our case, the total available capital stock increases due to investments; so, it is about capital *allocation over time*. Which variable guides the allocation decision?

In a private enterprise economy, the investment decision is induced by the rate of return differential between sectors. There may be two cases, $r_Y > r_X$ and $r_X > r_Y$, to begin with. One reason for such a difference may be due to the fact that the short-run relative price is out of sync with the long-run one; it is either high or low. The other reason may have to do with the initial composition of capital stocks. A developing economy usually begins with a small formal sector in terms of the output and capital stock than the informal sector. Hence, the historically given composition of sectoral capital stocks, K_Y/K_X, in the short run would be low to begin with. This leads to relatively higher return to the formal capital initially, giving us the first possibility $r_Y > r_X$.

The other possibility, $r_X > r_Y$, may arise due to the economic environment of the informal sector. The informal sector operates under a loose regulatory environment and can often avoid paying taxes. The minimum wage law is not binding and other benefits to workers are mostly absent. These may lead to a high return to informal-capital. Further, high real interest rates in the informal credit market is a well-known fact, which in turn implies a high rate of return to informal capital. The informal credit markets primarily serve the informal sector. See Chattopadhyay and Mondal (2017) for a detailed discussion on these possibilities.

With investment, the composition K_Y/K_X will change over time. Suppose that starting from a given composition, it so happens that $r_Y > r_X$. Figure 2 presents a temporary (short-run) equilibrium in which capital market is not in equilibrium. What are the effects on r_Y and r_X (and therefore on the return differential $r_Y - r_X$), sectoral wage rates, employment, and outputs if all investments go to the formal sector? If the returns are not equal in the short run it must be that the current price 'p' is either higher or lower than what is consistent with long-run p. So it is conceivable that a change in K_Y will also call for a simultaneous adjustment in 'p', more so because two sectors produce different varieties of goods.

With increase in K_Y, the gap $(r_Y - r_X)$ must close fully over finite time so that accumulation is stable.

3.1 Investment in the formal sector

Consider first a *ceteris paribus* effect of rising K_Y on other variables. A rise in K_Y causes $p.MPL_Y$ to shift right – L_Y increases, L_X decreases[17] and w_X increases. If p were constant, a higher L_Y along with higher K_Y would raise Y, and a lower L_X along with the same level of K_X would decrease X. This is the Rybczynski effect for the formal-informal economy, at a *constant* p. But this is not all; a rise in w_X leads to a rise in w_Y by equation (11). That in turn reduces L_Y. However, the reduction in L_Y cannot fully compensate the initial increase in it, for $0 < e_\alpha < 1$. At the same time, we do not know what happens to p – it may increase or decrease depending on the relative shifts in the demand and supply curves. If it increased, there would be a secondary rightward shift in $p.MPL_Y$ leading to a rise in L_Y. Therefore, the net effect on L_Y is ambiguous. To find that out, we turn to explicit equations of change. Our specific interest lies in finding out what happens to the ratio (r_Y/r_X) as K_Y increases.

3.2 Equations of change

Totally differentiating equations (1) and (2) and using cost minimisation conditions we get,

$$\theta_{\mathrm{LY}}\hat{w}_Y + \theta_{\mathrm{KY}}\hat{r}_Y = \hat{p}, \tag{13}$$

$$\theta_{\mathrm{LX}}\hat{w}_X + \theta_{\mathrm{KX}}\hat{r}_X = 0, \tag{14}$$

where θ_{ij} are the factor distributive shares, e.g. $\theta_{\mathrm{LX}} = w_{\mathrm{X}}\, a_{\mathrm{LX}}/P_{\mathrm{X}} = w_{\mathrm{X}}\, a_{\mathrm{LX}}$ (as P_{X} has been normalised to 1) and $\theta_{\mathrm{LY}} = w_{\mathrm{Y}}\, a_{\mathrm{LY}}/p$ with $\theta_{\mathrm{L}i} + \theta_{\mathrm{K}i} = 1$.

The CIH in value sense implies $\theta_{\mathrm{KY}}/\theta_{\mathrm{LY}} > \theta_{\mathrm{KX}}/\theta_{\mathrm{LX}}$. When labour market distortion is not deep, the initial wage premium is low, i.e. w_{Y} is set at a lower value leading to a smaller value of θ_{LY} ($\theta_{\mathrm{LY}} = w_{\mathrm{Y}}\, a_{\mathrm{LY}}/p$) and a larger value of θ_{KY}. Consequently $\theta_{\mathrm{KY}}/\theta_{\mathrm{LY}}$ would be larger. When w_{Y} is higher instead, labour share in income increases and the capital-share decreases ($\theta_{\mathrm{KY}} + \theta_{\mathrm{LY}} = 1$) and therefore $\theta_{\mathrm{KY}}/\theta_{\mathrm{LY}}$ decreases. So, it may so happen that though the CIH is satisfied in the physical sense ($K_{\mathrm{Y}}/L_{\mathrm{Y}} > K_{\mathrm{X}}/L_{\mathrm{X}}$), the CIH in a value sense gets reversed ($\theta_{\mathrm{KY}}/\theta_{\mathrm{LY}} < \theta_{\mathrm{KX}}/\theta_{\mathrm{LX}}$). We assume the values of w_{Y} to be such that the CIH in value sense always coincides with the CIH in the physical sense and investment in either sector *never reverses this relation*.

Manipulating equations (13) and (14) and using equation (12):

$$\theta_{\mathrm{KX}}\theta_{\mathrm{KY}}(\hat{r}_Y - \hat{r}_X) = (\theta_{\mathrm{KY}}\theta_{\mathrm{LX}} - \theta_{\mathrm{KX}}\theta_{\mathrm{LY}}e_\alpha)\hat{w}_X + \theta_{\mathrm{KX}}\hat{p} \tag{15}$$

As $0 < e_\alpha < 1$, and with the CIH, the coefficient of $\hat{w}_X = (\theta_{\mathrm{KY}}\theta_{\mathrm{LX}} - \theta_{\mathrm{KX}}\theta_{\mathrm{LY}}e_\alpha) > 0$.

Therefore, if $\hat{w}_X > 0$ and $\hat{p} > 0$, $\frac{r_Y}{r_X}$ would increase further, and if $\hat{w}_X < 0$ and $\hat{p} < 0$, $\frac{r_Y}{r_X}$ would decrease. For capital accumulation to be stable, $\frac{r_Y}{r_X}$ must decrease.

To figure this out, we turn to the demand conditions [equations (8) and (9)] and the full-employment of labour [equation (6)].

3.3 Equation of change involving full-employment

Using equations (3) and (4) into equation (6):

$$\frac{a_{LX}}{a_{KX}}K_X + \frac{a_{LY}}{a_{KY}}K_Y = \overline{L}.$$

Total differentiation leads to:

$$\lambda_{LX}[\hat{a}_{LX} - \hat{a}_{KX}] + \lambda_{LY}[\hat{a}_{LY} - \hat{a}_{KY}] + \lambda_{LY}\hat{K}_Y = 0.$$

Define the elasticity of labour's marginal physical products (or the elasticity of unit labour requirement with respect to its real price w) as:

$$\gamma_{LY} \equiv -\frac{\hat{a}_{LY} - \hat{a}_{KY}}{\hat{w}_Y - \hat{p}};\ \gamma_{LX} \equiv -\frac{\hat{a}_{LX} - \hat{a}_{KX}}{\hat{w}_X}.$$

Note that elasticities γ_{LX} and γ_{LY} are defined as positive for downward sloping marginal product curves.

Using these definitions, it follows that [and using equation (6)]:

$$\lambda_{LY}\gamma_{LY}\left[\hat{p} - e_{\alpha}\hat{w}_X\right] - \lambda_{LX}[\gamma_{LX}\hat{w}_X] + \lambda_{LY}\hat{K}_Y = 0$$

$$[\lambda_{LY}\gamma_{LY}e_{\alpha} + \lambda_{LX}\gamma_{LX}]\hat{w}_X - \lambda_{LY}\gamma_{LY}\hat{p} = \lambda_{LY}\hat{K}_Y \tag{16}$$

where $[\lambda_{LY}\gamma_{LY}e_{\alpha} + \lambda_{LX}\gamma_{LX}] > 0$ as $0 < e_{\alpha} < 1$. This is similar to the economy-wide average elasticity of labour demand. Because $e_{\alpha} < 1$, this average is less than the simple average.

3.4 Equation of change with non-homothetic preference

The demand-supply balance for X-goods implies [equation (8)]:

$$D_X = w_Y L_Y + w_X L_X = X.$$

Using equations (3) and (4):

$$\frac{a_{LX}}{a_{KX}} w_X K_X + \frac{a_{LY}}{a_{KY}} w_Y K_Y = \frac{K_X}{a_{KX}}.$$

Totally differentiating both sides (with constant level of K_X):

$$\theta_X[(\hat{a}_{LX} - \hat{a}_{KX}) + \hat{w}_X] + \theta_Y\left[(\hat{a}_{LY} - \hat{a}_{KY}) + \hat{w}_Y + \hat{K}_Y\right] = -\hat{a}_{KX}, \tag{17}$$

where θ_i is the labour income share in the total labour income of the ith sector and is a positive fraction.

By the envelope properties of cost minimisation we have:

$$w_Y da_{LY} + r_Y da_{KY} = 0$$

Writing the above in proportionate terms:

$$\theta_{LY}\hat{a}_{LY} + \theta_{KY}\hat{a}_{KY} = 0. \tag{18}$$

Now from the definition of γ_{LY} and using equation (18) it can be derived that:

$$\hat{a}_{KY} = \gamma_{LY}\theta_{LY}\hat{w}_Y. \tag{19}$$

$$\text{Similarly, } \hat{a}_{KX} = \gamma_{LX}\theta_{LX}\hat{w}_X. \tag{20}$$

Using equation (20) in equation (17) we get:

$$[\theta_X(1 - \gamma_{LX}) + \theta_Y(1 - \gamma_{LY})e_{\alpha} + \gamma_{LX}\theta_{LX}]\hat{w}_X + \theta_Y\gamma_{LY}\hat{p} = -\theta_Y\hat{K}_Y. \tag{21}$$

To determine the sign of the coefficient of $\hat{w}_X$, we need to know the likely values of γ_{LX} and γ_{LY}. Bhattacharya and Sakthivel (2005), by using NSSO data of India of various rounds, found that elasticity of employment to wage was −0.29 for 38th round (1983), −0.37 for 43rd round (1987-88), −0.66 for 50th round (1993-94) and −0.57 for 55th round (1999-00). In fact,

such an inelastic wage elasticity of employment is not peculiar to India. By citing other studies, they note that even for advanced countries such as Australia, the wage elasticity is −0.34. Following such findings, we may assume that γ_{LY} is inelastic ($0 < \gamma_{LY} < 1$). Further, between 1999 and 2007, there has been hardly any change in the level of informal employment in India (Chattopadhyay and Mondal, 2017; Karan and Selvaraj, 2008), though the real wage rate has gone up from Rs 16,000 to Rs 25,500 (Table I). This suggests that elasticity of employment to wage in the informal sector is also inelastic, i.e. $0 < \gamma_{LX} < 1$.

Therefore, the coefficient of $\hat{w}_X$ in (21) is positive. Eliminating $\hat{p}$ from equations (16) and (21), we get $\hat{w}_X = 0$. Consequently, $\hat{p} = -\frac{\hat{K}_Y}{\gamma_{LY}}$. Capital accumulation in the formal sector causes its relative price to fall. As γ_{LY} is a positive fraction, $|\hat{p}| > |\hat{K}_Y|$.

From equation (15), $\theta_{KX}\theta_{KY}(\hat{r}_Y - \hat{r}_X) = \theta_{KX}\hat{p} = -\frac{\theta_{KX}\hat{K}_Y}{\gamma_{LY}}$
i.e.:

$$(\hat{r}_Y - \hat{r}_X) = -\frac{\hat{K}_Y}{\gamma_{LY}\theta_{KY}^{KY}} < 0 \tag{22}$$

So $\frac{r_Y}{r_X}$ decreases with investments, and therefore accumulation is stable. This can also be seen from equations (13) and (14). As $\hat{w}_X = 0$, by equation (12), $\hat{w}_Y = 0$ and by equation (13), $\hat{r}_Y < 0$ and by equation (14), $\hat{r}_X = 0$. So starting with $(r_Y - r_X) > 0$, the gap closes. Further, as both w_Y and w_X do not change, the initial wage gap $(w_Y - w_X)$ does not close. Note that these results do not require any CIH[18]. The results are summarised below in P2.

Proposition 2. Consider investments in the formal sector, with non-homothetic preference and inelastic wage elasticities of employment in both sectors. Then:

- The capital returns equalise in finite time and the economy produces both varieties of goods without any change in output and employment composition.
- The formal-informal wage gap continues to persist.
- The result does not require a CIH to be satisfied.

An intuitive explanation can be given as follows. A rise in formal capital at constant p shifts the $pMPL_Y$ curve to the right, increases L_Y and decreases L_X. This increases the output of the formal sector and decreases the output of the informal sector. From Figure 3, the relative supply curve of X shifts left. At the same time, a fall in L_X causes w_X to rise so that the total wage bill increases, but the additional labour income is always spent on informal goods. So, the relative demand curve for X in Figure 3 shifts right. This increases $1/p$ or decreases p (adverse terms of trade effect). A fall in p causes a secondary leftward shift in the $pMPL_Y$ curve, so much so that the initial rightward shift due to a higher K_Y is exactly offset leaving the output and labour compositions unaffected – this is the demand side effect and highlights the role of endogenous demand. Without a demand side in the model, one would expect to observe a rise in the informal wage at a fixed relative price – a favourable effect on

Table I.
India – Average real wage rates (per year, at 2011 prices, in Rs '00)

Year	Formal Wage Rate	Informal Wage Rate	Gap
1994-1995	499	166	333
2000-2001	469	224	245
2005-2006	443	255	188
2010-2011	467	255	212

informal workers that is absent when relative prices are allowed to adjust. As the position of $pMPL_Y$ is not affected by capital accumulation, the initial wage gap, output and labour composition all remain unchanged.

Next, as w_X does not change, from equation (14), r_X does not change as well so that (w_X/r_X) remains the same. Hence, (K_X/L_X) must remain the same. As K_X is fixed, L_X cannot change. This means that L_Y also remains fixed – the labour composition remains unaffected. However, K_Y increases so that (K_Y/L_Y) increases. Therefore (w_Y/r_Y) must increase. But w_Y does not change, so r_Y must decrease. With unchanged r_X, the gap $(r_Y - r_X)$ must close over time. Though the ratio (K_Y/L_Y) increases – which should have increased (Y/L_Y) if p were to remain fixed – the formal sector's output does not change because of a sharp fall in the relative price. The informal sector's output does not change as K_X/L_X remains the same. Therefore (Y/X) ratio remains the same.

From above, the following corollary is immediate.

Corollary 1

Investments in the formal sector do not affect the formal and informal wages, and output and employment composition due to an offsetting rise in the relative price of the informal goods.

We now turn to the homothetic-preference case.

3.5 Equation of change with homothetic preferences

Writing equation (9) in terms of proportionate changes:

$$\hat{Y} - \hat{X} = -\sigma_D \hat{p}, \tag{23}$$

where σ_D is the elasticity of substitution (Y- for X-goods) in demand and is defined as positive for a downward sloping demand curve.

Using equations (3) and (4):

$$\hat{K}_Y - \hat{a}_{KY} - \hat{K}_X + \hat{a}_{KX} = -\sigma_D \hat{p}. \tag{24}$$

Using equations (19) and (20) in equation (24), $(\hat{K}_X = 0)$:

$$\hat{K}_Y - \gamma_{LY}\theta_{LY}\hat{w}_Y + \gamma_{LX}\theta_{LX}\hat{w}_X = -\sigma_D \hat{p}$$

$$[\gamma_{LX}\theta_{LX} - \gamma_{LY}\theta_{LY}e_\alpha]\hat{w}_X + \sigma_D \hat{p} = -\hat{K}_Y$$

$$B\hat{w}_X + \sigma_D \hat{p} = -\hat{K}_Y \tag{25}$$

where $B \equiv [\gamma_{LX}\theta_{LX} - \gamma_{LY}\theta_{LY}e_\alpha]$.

Now, $B > 0$ if $[\gamma_{LX}\theta_{LX} - \gamma_{LY}\theta_{LY}e_\alpha] > 0$. As the informal sector is labour intensive, $\theta_{LX} > \theta_{LY}$, so the above condition is definitely met when $\gamma_{LX} > \gamma_{LY}\,(0 < e_\alpha \leq 1)$ – the MPL curve of the formal sector must be more inelastic (recall that both MPL_Y and MPL_X are inelastic by assumption) than that of the informal sector.

Rewrite equation (16) as:

$$A\hat{w}_X - \lambda_{LY}\gamma_{LY}\hat{p} = \lambda_{LY}\hat{K}_Y \tag{16A}$$

where $A \equiv [\lambda_{LY}\gamma_{LY}e_\alpha + \lambda_{LX}\gamma_{LX}] > 0$.

The determinant of the coefficient matrix for the system of equations (16A) and (25) is given by:

$$D = \begin{vmatrix} A & -\lambda_{LY}\gamma_{LY} \\ B & \sigma_D \end{vmatrix}$$

$$D = A\sigma_{\mathrm{D}} + B\lambda_{\mathrm{LY}}\gamma_{\mathrm{LY}} > 0$$

Solving equations (16A) and (25) by Cramer's rule:

$$\hat{w}_X = \frac{\lambda_{LY}(\sigma_D - \gamma_{LY})}{[\sigma_D A + B\lambda_{LY}\gamma_{LY}]}\hat{K}_Y$$

Formal goods (durables) are likely to be more price elastic than informal (non-durables), so $\sigma_{\mathrm{D}} > 1$[19]. As $\gamma_{\mathrm{LY}} < 1$, the numerator is > 0 implying $\hat{w}_X > 0$ – with elastic (or unit elastic) relative demand for goods and inelastic labour demand, investment in the formal sector increases the informal wage.

The solution for p is given by,

$$\hat{p} = -\frac{A + B\lambda_{LY}}{[\sigma_D A + B\lambda_{LY}\gamma_{LY}]}\hat{K}_Y.$$

The numerator is positive as A, B, and $\lambda_{\mathrm{LY}} > 0$. Therefore $\hat{p} < 0$ (like the previous case). Now in equation (15), the coefficient of $\hat{w}_X > 0$. Therefore, the effect on the return gap is ambiguous as one component (wage) on the right-hand side (RHS) of equation (15) increases, but the other (relative price) decreases. From equations (13) and (14), as $\hat{p} < 0$ and $\hat{w}_Y > 0$, $\hat{r}_Y < 0$ and as $\hat{w}_X > 0$, $\hat{r}_X < 0$. So both r_{Y} and r_{X} decrease. For the gap $(r_{\mathrm{Y}} - r_{\mathrm{X}})$ to close r_{Y} should fall faster than r_{X}, i.e. $\frac{\hat{r}_X}{\hat{r}_Y} > 1$ – so that capital rates get equalised across the sectors in finite time as K_{Y} rises.

With the CIH, we have $\theta_{\mathrm{KY}}\,\theta_{\mathrm{LX}}/\theta_{\mathrm{KX}}\theta_{\mathrm{LY}} > 1$. With $0 < e_\alpha \leq 1$, the condition can also be written as $\frac{\theta_{KY}\theta_{LX}}{\theta_{KX}\theta_{LY}(1+e_\alpha)} > 1$.

Eliminate w_{Y} and w_{X} from equations (13) and (14) to get:

$$\theta_{\mathrm{KY}}\theta_{\mathrm{LX}}\hat{r}_Y - \theta_{\mathrm{KX}}\theta_{\mathrm{LY}}e_\alpha\hat{r}_X = \theta_{\mathrm{LX}}\hat{p}. \tag{26}$$

As $\hat{p} < 0$, $\theta_{\mathrm{KY}}\theta_{\mathrm{LX}}\hat{r}_Y - \theta_{\mathrm{KX}}\theta_{\mathrm{LY}}e_\alpha\hat{r}_X < 0$.
So, $\frac{\hat{r}_X}{\hat{r}_Y} > \frac{\theta_{KY}\theta_{LX}}{\theta_{KX}\theta_{LY}e_\alpha} > 1$.

Therefore, starting from $r_{\mathrm{Y}} > r_{\mathrm{X}}$, the gap closes over time. So it must be that $(\hat{r}_Y - \hat{r}_X) < 0$ in equation (15). The question now is whether the wage gap closes before the return gap does. Otherwise the formal-informal wage gap would continue to exist.

From equation (15), the net effect of a rising w_{X} and a falling p is to decrease the gap $(r_{\mathrm{Y}} - r_{\mathrm{X}})$. Now the rate of rise in w_{X} would be faster than the rate of fall in the return gap when the value of the coefficient $\left[\frac{\theta_{KY}\theta_{LX} - \theta_{KX}\theta_{LY}e_\alpha}{\theta_{KX}\theta_{KY}}\right]$ of $\hat{w}_X$ and $\hat{w}_X$itself are larger at any *given* $\hat{p}$. This means $\left[\frac{\theta_{KY}\theta_{LX} - \theta_{KX}\theta_{LY}e_\alpha}{\theta_{KX}\theta_{KY}}\right] = \left[\frac{\theta_{LX}}{\theta_{KX}} - \frac{\theta_{LY}}{\theta_{KY}}e_\alpha\right]$ should be larger. This is likely to happen when the initial wage premium is smaller (leading to a smaller value of 'α') to begin with[20]. Further, $\hat{w}_X$is larger when σ_D is larger and γ_{LY} is smaller, i.e. the relative demand is elastic

and the wage elasticity of formal-employment is sufficiently inelastic. Therefore, with a smaller wage premium and elastic relative demand, the return gap would close slowly, giving way for a possibility of wages getting equalised before the closing of the return gap [21]. This is intuitive – when the economy begins with a smaller wage gap, the possibility of wage gap closing completely before the closing of return gap is more.

Such a possibility is shown in Figure 4. Nevertheless, the wage gap decreases over time even if it does not close fully, unlike the case with non-homothetic preferences in which the initial wage gap does not change at all.

Irrespective of wage equalisation, output of the formal sector expands (with higher K_Y and L_Y) and output of the informal sector contract contracts (with same K_X and lower L_X), so that the (Y/X) ratio increases. In this case, a reduction in p does not fully offset the effect of higher K_Y. At a constant p, by Rybczynski's theorem, a higher K_Y should expand the formal sector and should contract the informal sector, i.e. formal-to-informal output ratio should increase. Hence, Rybczynski's theorem still holds even with a variable p.

3.6 Behaviour of return gap after wage equalisation

When wages equalise, does the return gap still continue to decrease and close eventually? The equations of change with $w_Y = w_X = w$ modifies to:

$$\theta_{LY}\hat{w} + \theta_{KY}\hat{r}_Y = \hat{p} \tag{27}$$

$$\theta_{LX}\hat{w} + \theta_{KX}\hat{r}_X = 0 \tag{28}$$

Eliminating w from equations (27) and (28) we get:

$$\theta_{LX}\hat{p} = \theta_{KY}\theta_{LX}\hat{r}_Y - \theta_{KX}\theta_{LY}\hat{r}_X. \tag{29}$$

First we need to know what happens to p and w after wage equalisation. In equation (16), put $e_\alpha = 1$ to get:

$$[\lambda_{LY}\gamma_{LY} + \lambda_{LX}\gamma_{LX}]\hat{w} - \lambda_{LY}\gamma_{LY}\hat{p} = \lambda_{LY}\hat{K}_Y$$

$$\lambda\hat{w} - \lambda_{LY}\gamma_{LY}\hat{p} = \lambda_{LY}\hat{K}_Y, \tag{30}$$

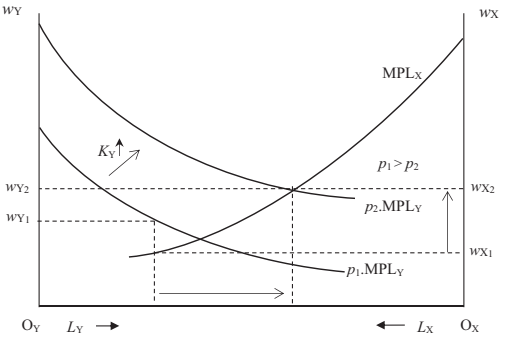

Figure 4.
Wage equalisation with homothetic preference

where $\lambda \equiv \lambda_{LY}\gamma_{LY}+\lambda_{LX}\gamma_{LX} > 0$ can be termed as the economy-wide weighted average value of the marginal products of labour.

Equation (25) modifies to:

$$B\hat{w} + \sigma_D\hat{p} = -\hat{K}_Y \tag{31}$$

where $B \equiv \gamma_{Lx}\theta_{LX} - \gamma_{LY}\theta_{LY}$.

Like the previous case, $B > 0$ as the *MPL* curve of the formal sector is more inelastic than that of the informal sector.

Applying Cramer's rule to equations (30) and (31), the solution for p is given by:

$$\hat{p} = -\frac{\lambda + B\lambda_{LY}}{[\sigma_D\lambda + B\lambda_{LY}\gamma_{LY}]}\hat{K}_Y.$$

Both the numerator and denominator are positive, so $\hat{p} < 0$.

The solution for w is given by:

$$\hat{w} = \frac{\lambda_{LY}(\sigma_D - \gamma_{LY})}{[\sigma_D\lambda + B\lambda_{LY}\gamma_{LY}]}\hat{K}_Y.$$

As $\sigma_D > 1$ and $\gamma_{LY} < 1$, the numerator is positive. Hence, $\hat{w} < 0$.

Therefore, from equations (27) and (28), both r_Y and r_X continue to decrease. So for the closing of the return gap, (r_X/r_Y) should rise.

From equation (29), as $\hat{p} < 0$, $\theta_{KY}\theta_{LX}\hat{r}_Y - \theta_{KX}\theta_{LY}\hat{r}_X < 0$, i.e. $\frac{\hat{r}_X}{\hat{r}_Y} > \frac{\theta_{KY}\theta_{LX}}{\theta_{KX}\theta_{LY}} > 1$s the formal sector is capital-intensive. The return gap will close in finite time and the economy will produce both varieties of goods (non-specialisation) with the formal sector occupying most of the total gross domestic product (GDP) and employment.

The following proposition summarizes the results obtained in Sections 3.5 and 3.6.

Proposition 3. Consider investments in the formal sector with homothetic preferences, a capital-intensive formal sector and inelastic wage elasticities of employment in both the sectors. Then:

(1) The capital returns equalise in finite time and the economy produces both varieties of goods, irrespective of the closing of the formal-informal wage gap.

(2) The formal sector expands and the informal sector contracts in terms outputs – Rybczynski's theorem holds with variable relative price.

(3) The formal-informal wage gap closes only if:

- the wage premium for the formal sector is not too high to begin with; and
- the relative demand between formal and informal goods is elastic and the wage elasticity of formal-employment is sufficiently inelastic.

The difference in the results with the non-homothetic case lies with the behaviour of the relative price. With homothetic preferences, part of the additional expenditure falls on formal goods as well, so that the downward pressure on the relative price is less. As a result, the favourable effect of investment in the formal sector on the informal wage is not fully offset. This observation leads to the following corollary.

Corollary 2. Though investments in the formal sector increases the relative price of the informal goods, it has favourable effects on both formal and informal wages.

4. Investment in the informal sector

Now we consider the other possibility that, starting from a given (K_Y/K_X), it happens that $r_X > r_Y$. The relevant question now is what are the effects on r_Y and r_X (and therefore on the return differential $r_X - r_Y$), sectoral wage rates, employment and outputs if in this period the *entire* output of investment goods is directed to the informal sector to increase K_X?

An increase in K_X increases MPL_X at the initial L_X, i.e. shifts the MPL_X curve up. From the labour allocation diagram, L_Y remains the same at the initial p and w_Y and therefore L_X also remains the same, accompanied by a rise in w_X[22]. A rise in w_X causes w_Y to increase by equation (11), but less that proportionately. A rise in w_Y causes L_Y (L_X) to decrease (increase) now, so the shift in the supply curve of X-goods depends on the net effect of these changes. On the other hand, as both w_X and w_Y are rising, the aggregate wage bill increases and that shifts the demand curve to the right. So to figure out the effects on wages and relative prices, let us now look at the equations of change with increasing K_X. Note that there are no changes in the equations (13) and (14) and therefore in equation (15).

4.1 Equation of change with full employment

The full-employment condition $a_{LY}\,Y + a_{LX}\,X = \overline{L}$ in terms of proportionate change now reads:

$$\lambda_{LX}[\hat{a}_{LX} - \hat{a}_{KX}] + \lambda_{LY}[\hat{a}_{LY} - \hat{a}_{KY}] + \lambda_{LX}\hat{K}_X = 0.$$

Following same procedure as in Section 3.3, we get:

$$A\hat{w}_X - \lambda_{LY}\gamma_{LY}\hat{p} = \lambda_{LX}\hat{K}_X \tag{32}$$

and, as before, $A \equiv [\lambda_{LY}\gamma_{LY}e_\alpha + \lambda_{LX}\gamma_{LX}] > 0$.

4.2 Equation of change with non-homothetic preferences

Equation (17) modifies to:

$$\theta_X\left[(\hat{a}_{LX} - \hat{a}_{KX}) + \hat{w}_X + \hat{K}_X\right] + \theta_Y[(\hat{a}_{LY} - \hat{a}_{KY}) + \hat{w}_Y] = \hat{K}_X - \hat{a}_{KX}.$$

Using, $\hat{a}_{KX} = \gamma_{LX}\theta_{LX}\hat{w}_X$,

$$[\theta_X(1 - \gamma_{LX}) + \theta_Y e_\alpha(1 - \gamma_{LY}) + \gamma_{LX}\theta_{LX}]\hat{w}_X + \theta_Y\gamma_{LY}\hat{p} = (1 - \theta_X)\hat{K}_X$$

$$B\hat{w}_X + \theta_Y\gamma_{LY}\hat{p} = (1 - \theta_X)\hat{K}_X \tag{33}$$

As both γ_{LX} and γ_{LY} are inelastic, $B > 0$.

Solving equations (32) and (33) by Cramer's rule:

$$\hat{w}_X = \frac{\theta_Y}{A\theta_Y + B\lambda_{LY}}\hat{K}_X.$$

Both the numerator and denominator are positive, so $\hat{w}_X > 0$, and

$$\hat{p} = \frac{A(1 - \theta_X) - B\lambda_{LX}}{A\theta_Y\gamma_{LY} + B\lambda_{LY}\gamma_{LY}}\hat{K}_X.$$

Now, starting with $(r_X - r_Y) > 0$, accumulation is stable when r_Y/r_X increases, *i.e.* $\frac{\hat{r}_Y}{\hat{r}_X} > 1$. From equation (26), $\frac{\hat{r}_Y}{\hat{r}_X} > 1$ only if $\hat{p} > 0$. Note also from equation (15) that with $\hat{w}_X > 0$, $\frac{\hat{r}_Y}{\hat{r}_X} > 1$ when $\hat{p} > 0$. The relative price of informal goods rises when the rightward shift in the relative supply curve of informal goods (in Figure 3) is more than the demand for it. Additional income generated due to investments in the informal sector is paid to capital-owners and workers. But only the workers spend on informal goods and their income is always a fraction of total income (= value of production in equilibrium). Hence, relative supply shifts to the right more than relative demand does, so that $1/p$ decreases or p increases. Therefore, the numerator $A(1 - \theta_X) - B\lambda_{LX} > 0$ and the return gap closes over time.

Does the wage gap close before the return gap does? From equation (15) we have:

$$(\hat{r}_Y - \hat{r}_X) = \frac{\theta_{KY}\theta_{LX} - \theta_{KX}\theta_{LY}e_\alpha}{\theta_{KX}\theta_{KY}}\hat{w}_X + \frac{\hat{p}}{\theta_{KY}}.$$

The RHS is a weighted average of $\hat{w}_X$ and $\hat{p}$ (both > 0). The weight on $\hat{p}$ is $1/\theta_{KY} > 1$. If the other weight (on $\hat{w}_X$) $\left[\frac{\theta_{KY}\theta_{LX} - \theta_{KX}\theta_{LY}e_\alpha}{\theta_{KX}\theta_{KY}}\right] > 1$, then $(\hat{r}_Y - \hat{r}_X) > \hat{w}_X$ and the return gap closes before wages. We have already deduced that this weight is larger when the initial wage premium is smaller. Here both w_X and p rise, so that the return gap closes faster and consequently the capital rates equalise before wages. Though the wage gap reduces to some extent, *it continues to persist in the long run* – see Figure 5 below.

Looking at Figure 5, one can figure out the behaviour of sectoral output, employment composition and factor returns. First note that, due to strict convexity of the union's indifference map, a rise in p also raises L_Y (wage and employment are both an economic 'good'). With fixed K_Y, (K_Y/L_Y) decreases, so (w_Y/r_Y) must decrease as well. As $\hat{w}_Y > 0$, it must be that $\hat{r}_Y > \hat{w}_Y > 0$. From equation (13), we get $\hat{r}_Y > \hat{p} > \hat{w}_Y -$ this is the well-known magnification effect due to a change in p (Jones, 1965). From equation (14), as $\hat{w}_X > 0$, $\hat{r}_X < 0$. So, starting with $(r_X - r_Y) > 0$, the return gap would close fully. As L_Y increases, the formal sector's output rises. As K_X increases, but L_X decreases, the effect on the informal sector's output seems ambiguous, but it can be shown that output of the informal sector also increases – see Appendix 2. The favourable effect on the formal output due to investments in the informal sector is due to a better relative price of the formal good. At a constant p, by Rybczynski's theorem, higher K_X should expand the informal sector and should contract the formal sector, i.e. the formal-to-informal output ratio should decrease.

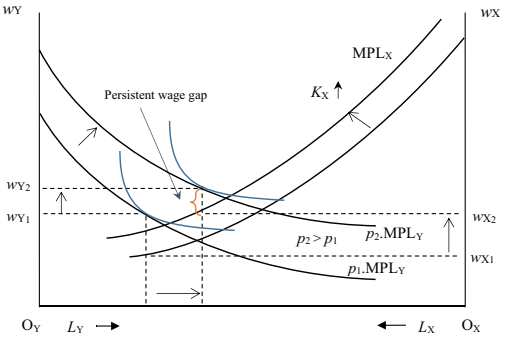

Figure 5.
Persistence of wage gap

However, the effect on the formal-to-informal output ratio cannot be ascertained here – it may increase or decrease depending on the extent of increase in the relative price, so that Rybczynski's theorem may not hold.

Compare these results with the case of investment in the formal sector (Section 3.4) with non-homothetic preferences. First, here *both* w_X and p *increase*. The effect of a rise in K_X on w_X is reinforced by a rise in p, so that the net effect of investments in the informal sector is to raise wages. Second, while the wage gap is *static* with investment in the formal sector, here the wage gap *reduces*, though does not close completely. Third, the formal-to-informal employment ratio increases in the present case.

Proposition 4. Consider investments in the informal sector, with non-homothetic preference and inelastic wage elasticities of employment in both sectors. Then:

- The capital returns equalise in finite time and the economy produces both varieties of goods, with a higher output and employment in the formal sector.
- Rybczynski's theorem may or may not hold (in terms of output ratio).
- The formal-informal wage gap reduces but does not close completely.
- The result requires a CIH to be satisfied.

The last result may be contrasted to P2(iii).

4.3 Equation of change with homothetic preferences

In equation (24), put $\hat{K}_Y = 0$ to get:

$$-\hat{a}_{KY} - \hat{K}_X + \hat{a}_{KX} = -\sigma_D \hat{p}$$

$$\left[\gamma_{LX}\theta_{LX} - \gamma_{LY}\theta_{LY}(1+e_\alpha)\right]\hat{w}_X + \sigma_D \hat{p} = \hat{K}_X$$

$$B\hat{w}_X + \sigma_D \hat{p} = \hat{K}_X \tag{34}$$

where $B > 0$ as before.

Solving equations (32) and (34) by Cramer's rule:

$$\hat{w}_X = \frac{\lambda_{LX}\sigma_D + \lambda_{LY}\gamma_{LY}}{A\sigma_D + B\lambda_{LY}\gamma_{LY}}\hat{K}_X.$$

Both the numerator and denominator are positive, hence $\hat{w}_X > 0$, and:

$$\hat{p} = \frac{A - B\lambda_{LX}}{A\sigma_D + B\lambda_{LY}\gamma_{LY}}\hat{K}_X.$$

Now, investment in the informal sector shifts the relative supply of informal goods but the relative demand curve does not shift; hence $1/p$ must decrease. So the numerator of the above expression must be positive and $\hat{p} > 0$. Note that in the case of non-homothetic preference we assumed $A(1-\theta_X) - B\lambda_{LX} > 0$. As $(1-\theta_X) < 1$, in effect, we are assuming that the labour-income share in the formal sector ($\theta_Y = 1 - \theta_X$) is not too low[23], so that even when 'A' is multiplied by a number less than one, we still get $A(1-\theta_X) - B\lambda_{LX} > 0$.

Therefore, with homothetic preferences too, given that the initial wage premium is not large, wages are unlikely to equalise due to investments.

5. Concluding remarks

This paper has tried to understand the temporal effect of investments on the formal-informal wage gap – a typical labour market feature observed in developing economies – in a general equilibrium set up. We ask a specific question: would the wage gap close over the long run and could investment be a driver of such convergence? The exercises carried out are therefore best viewed as theoretical experiments.

The model has introduced a particular variety of a labour market distortion where the formal wage is determined by union bargaining, but has stayed away from other types of labour market segmentation such as differences in skill levels among workers or tax considerations where only the formal sector pays the tax. The idea is to capture only one type of heterogeneity among many – where the source of heterogeneity lies in the form of a bargained nominal wage in the formal sector – and then to investigate what happens to the wage gap over time due to investments going either to the formal or to the informal sector. Union wage bargaining is the common means of wage determination in developing (Marjit and Kar, 2011) and developed economies (Farber, 1986; Lingens, 2007).

The analyses show that with a capital-intensive formal sector, a small initial wage premium and elastic relative demand, investment in the formal sector is likely to close the wage gap if the preferences are homothetic, but the gap persists if the preferences are non-homothetic. However, investment in the informal sector is unlikely to close the wage gap with either type of preferences. Though we have assumed that workers do not save, the results would hold so long as the saving propensity of the capitalists is higher than that of workers. For developing economies like India, the preference over goods is likely to be non-homothetic (as noted in Section 2.3), and therefore – though the wage gap seems to have narrowed – the wage gap is likely to persist over time, irrespective of which sector is getting investments.

In a similar general equilibrium framework, but with fixed (nominal) formal wage and fixed relative prices, Chattopadhyay and Mondal (2017) find that investment in the formal sector equalises wages, but the gap persists with investment in the informal sector. Our results may also be compared with Santra (2014), who explores the impact of trade on the relative returns to skill in the presence of non-homothetic preferences. He shows that, in a general equilibrium model of trade in differentiated products, if consumers react to a rise in real income by increasing their demands of the relatively skill intensive commodities, then the skilled–unskilled wage ratio increases. Our study finds that a narrowing wage gap can possibly be explained by homothetic preferences.

Our study, though draws motivation from the study of convergence, is also different from it. Convergence is driven by capital accumulation depending on how far a country is from its steady state (conditional convergence). In our case, it is the difference in capital returns across sectors that drives accumulation, and in the process, wages may or may not converge. Further, an important property of the steady state (to which a country converges) is balanced growth. However, our model can have unbalanced growth even when the rates of capital returns are equalised.

The fact that investment seems to have different effects on the wage gap may offer us a few insights to investment policy in India. The recent slowdown in the GDP growth rate of India has been attributed to investment – investment expenditure declined faster than other components of aggregate demand and gross capital formation as percentage of GDP came down from 38.2 per cent in 2011-12 to 32.3 per cent in 2013-14 (Economic

Survey of India, various years). It is then not surprising that policy would call for more investment in the economy. Given the results of our model, and if one of the objectives of investment is to minimise the wage gap, it is straightforward that investments should be directed towards the formal sector. Various rounds of economic surveys emphasise growth through public investment such as investment in infrastructure and railways. Public investments can mostly be treated as investments going to the formal sector, and our model points out significant spillover effects to informal sector in terms of a narrowing wage gap over time.

Finally, the novelty of our study lies in taking a step towards building general equilibrium models with endogenous relative prices to investigate *temporal* effects of investments. It would be an interesting exercise to see the implications of incorporating both a unionised formal sector and heterogeneity in the skill levels in an optimising intertemporal multi-sector framework, much in the line of what García-Peñalosa and Turnovsky (2005) and Turnovsky and Basher (2009) do. We leave this for future research.

Notes

1. There are potentially other causes. For instance, a structural shift like economic reforms in the 1990s or the effects of central pay commissions on formal wages. In our study, all the NSSO rounds are post-reform rounds, hence, the effect of structural shift is already embedded. The effect of pay commission (like the sixth/seventh pay) is a *one-time* change (a level effect), and second, unless NSSO data (latest round being 2011) are available concurrently, it would be difficult to get a discernible trend.
2. Formal wage is obtained from the Annual Survey of Industries, India data. It is calculated from the Principal Characteristics of Factories by dividing wages to worker by number of workers. The data are on an annual basis and the deflator used is CPI-IW.
3. For calculating the annual average of real wages in the informal manufacturing sector, we have considered Own account Enterprises, Non Directory Manufacturing Enterprises and Directory Manufacturing Enterprises of the urban informal sector.
4. It may be noted here that Table I and Figure 1 just illustrate the fact that the wage gap in India does exist. But our objective in the paper is not to strictly relate this observation to the question of wage-convergence or to claim that results of our model spelt out later can potentially explain such observation in the past. Our question is purely a theoretical one that seeks to find out – given that there exists wage gap – whether investment across sectors could close the wage gap in the future (i.e. over time). In that sense, our question is not India-specific, but is more of an analytical in nature. Moreover, it is also possible that the wage gap might have increased further in the recent years – which can only be ascertained when the recent NSSO round is available.
5. The coefficient effect explains the contribution of the difference in returns to human capital variables and individual characteristics between formal and informal workers to the wage gap. The endowment effect explains the contribution of the differences in characteristics between formal and informal workers to the wage gap.
6. There is strong evidence of skilled-unskilled wage gap getting widened over the past few decades all over the world including developing economies like India as well (Santra, 2014). If the average wage gap is decreasing in India, increasing skilled–unskilled wage gap implies that the unskilled wage gap must be decreasing. This claim of course requires rigorous econometric exercise – which is possibly would be an extension of Narayanan's study to other rounds of NSSO.
7. All such general equilibrium model, in turn, are variations of Jones (1965).
8. The evidence for inter-sectoral (here formal-informal) mobility of capital is weak. La Porta and Shleifer (2008) find that informal firms rarely become formal; an average informal firm remains

in business for nearly a decade without attempting to become formal. Further, on average, 91 per cent of registered firms start out as registered. See also Marjit and Kar (2009) for the Indian context.

9. There has been attempt to estimate the size of so-called black economy (for India, see Kumar, 2016), the definition of which is somewhat different from a shadow economy. A shadow economy is an extra-legal entity that contributes to productive activities of an economy. Whether a black economy does so is debatable.

10. Contrast this definition of 'long run' to that found in the growth literature, where long run has more to do with the question of 'balanced growth' – a steady state in which all the relevant macro-variables grow at the same rate. With fixed labour endowment in our model, growth cannot be balanced. Our focus is more on whether wages equalise when capital market reaches equilibrium due to investments. However, the source of diminishing return is often the 'raw' labour even if one allows growth in labour force.

11. Homogeneous of degree one utility functions satisfy the required properties of homothetic preferences. It is well known that the standard utility maximisation with homogeneous of degree one function leads to demand functions that depend on relative prices, but not on absolute prices. This property is known as the 'homogeneity postulate'.

12. Such saving behaviour is not uncommon in the growth literature, for instance, see Uzawa (1961). It is also well known that qualitative results do not get affected due to such assumptions so long as the saving propensity of the capitalists exceeds that of workers.

13. Market clearing for the formal goods is given by, $pY = \psi\ (r_Y\ K_Y + r_X\ K_X)$ and that for investment goods by $p_I Y = (1 - \psi)\ (r_Y K_Y + r_X K_X)$, where ψ is the fraction of capitalists' income spent on Y-goods. The relative price of investment goods, p_I, is exogenously fixed.

14. Strictly speaking, such preference is quasi-homothetic. For workers, investment and formal goods and for capitalists, informal goods can be (loosely) treated as neutral goods, respectively.

15. While the wage premium is defined with respect to the market clearing wage, the wage gap is the difference between the "equilibrium" wages.

16. Note that the formal wage may rise and formal employment may actually fall under general equilibrium. But this exercise is for deriving the demand schedule.

17. That is, labour migrates from the informal to the formal sector.

18. Note that CIH and elasticities of marginal products of labour enter into the equations through the coefficients of$\hat{w}_X$. Further, that the CIH may not always matter is also shown by Chaudhuri (2008) – though in a different context – where he shows that the consequences of international factor mobility on wage inequality may not necessarily depend on the difference in the factor intensity condition.

19. If the Cobb-Douglas preferences are homogeneous of degree one then $\sigma_D = 1$.

20. We have already noted that with a smaller initial wage premium, w_Y is set at a lower value so that θ_{LY} is smaller and θ_{KY} is larger, leading to a smaller value of θ_{LY}/θ_{KY}.

21. With inelastic adjustment in 'α', we can treat w_Y to be 'as if' fixed so that an increase in w_X reduces the wage gap.

22. Note that, unlike the case with investments in the formal sector, in this case there is no 'reverse' (formal to informal) migration.

23. In India, for instance, GVA in the formal sector is close to 50 per cent of GDP. If one applies the standard two-third share of labour-income to GVA, it comes to 33 per cent of GDP. See Figure 1 in La Porta and Shleifer (2014) for a cross-country comparison of value added in the formal with the informal sector.

References

Acharyya, R. (2011), "A note on quantitative trade restrictions, income effects and wage inequality", *Economic Modelling*, Vol. 28 No. 6, pp. 2628-2633.

Agenor, P. (1996), "The labour market and economic adjustment", *IMF Staff Papers*, Vol. 43 No. 2, pp. 261-355.

Agenor, P.R. and Montiel, P.J. (2008), *Development Macroeconomics*, 3rd edition, Princeton University Press, Princeton, NJ.

Bargain, O. and Kwenda, P. (2009), "The informal sector wage gap: new evidence using quantile estimations on panel data", Discussion Paper No. 4286, Institute for the Study of Labor, Bonn.

Barro, R.J. and Sala-i-Martin, X. (2003), *Economic Growth*, MIT Press, MA.

Basu, K. (2013), *Analytical Development Economics: The Less Developed Economy Revisited. New*, Oxford University Press, Delhi.

Bhattacharya, B.B. and Sakthivel, S. (2005), "Employment, wage and output relationships in India: a comparison of pre and post reform behaviour", *The Indian Journal of Labour Economics*, Vol. 48 No. 2, pp. 243-258.

Carruth, A.A. and Oswald, A.J. (1981), "The determination of union and non-union wage rates", *European Economic Review*, Vol. 16 No. 2, pp. 285-302.

Chattopadhyay, S. and Mondal, R. (2017), "Characterisation of economic growth in developing economies with informal sector", *Economic Papers: A Journal of Applied Economics and Policy*, Vol. 36 No. 1, pp. 86-101.

Chaudhuri, S. (2008), "Wage inequality in a dual economy and international mobility of factors: do factor intensities always matter?", *Economic Modelling*, Vol. 25 No. 6, pp. 1155-1164.

Chaudhuri, S. (2003), "How and how far to liberalize a developing economy with informal sector and factor market distortions", *Journal of International Trade and Economic Development*, Vol. 12 No. 4, pp. 403-428.

Chaudhuri, S. (2016), "Trade unionism and welfare consequences of trade and investment reforms in a developing economy", *Metroeconomica*, Vol. 67 No. 1, pp. 152-171.

Chaudhuri, S. and Banerjee, D. (2007), "Economic liberalization, capital mobility and informal wage in a small open economy: a theoretical analysis", *Economic Modelling*, Vol. 24 No. 6, pp. 924-940.

Chaudhuri, S. and Banerjee, D. (2010), "Foreign capital inflow, skilled–unskilled wage inequality and unemployment of unskilled labour in a fair wage model", *Economic Modelling*, Vol. 27 No. 1, pp. 477-486.

Chaudhuri, S. and Biswas, A. (2016), "Endogenous labour market imperfection, foreign direct investment and external terms-of-trade shocks in a developing economy", *Economic Modelling*, Vol. 59, pp. 416-424.

Chaudhuri, S. and Yabuuchi, S. (2007), "Economic liberalization and wage inequality in the presence of labour market imperfection", *International Review of Economics and Finance*, Vol. 16 No. 4, pp. 592-603.

Daza, N. and Gamboa, L.F. (2013), "Informal-formal wage gaps in Colombia", Society for the Study of Economic Inequality, Working Paper 301.

Farber, H.S. (1986), "The analysis of union behaviour", in *Handbook of Labor Economics*, North Holland, Amsterdam, Vol 2, pp 1039-1089.

Fields, A. (1990), "Labour market modeling and the urban informal sector: theory and evidence", in Turnham, D. (Ed.), *The Informal Sector and Evidence Revisited*, OECD, Paris.

Frey, B. and Schneider, F. (2000), "Informal and underground economy (no. 0004)", Working Paper, Department of Economics, Johannes Kepler University of Linz.

García-Peñalosa, C. and Turnovsky, S.J. (2005), "Second-best optimal taxation of capital and labor in a developing economy", *Journal of Public Economics*, Vol. 89 Nos 5/6, pp. 1045-1074.

Global Findex/Financial Inclusion Data (2014), The World Bank, Washington, DC.

Gupta, M.S. and Dutta, P.B. (2011), "Skilled–unskilled wage inequality and unemployment: a general equilibrium analysis", *Economic Modelling*, Vol. 28 No. 4, pp. 1977-1983.

Heijdra, B.J. (2009), *Foundations of Modern Macroeconomics*, 2nd edition, Oxford University Press, New York.

Herrendorf, B., Rogerson, R. and Valentinyi, A. (2014), "Growth and structural transformation", in Aghion, P. and Durlauf, P. (Eds), *Handbook of Economic Growth*, Elsevier, Oxford, North Holland, Vol 2.

Jones, R.W. (1965), "The structure of simple general equilibrium models", *The Journal of Political Economy*, Vol. 73 No. 6, pp. 557-572.

Jones, R.W. (1971), "Distortions in factor markets and the general equilibrium model of production", *The Journal of Political Economy*, Vol. 79 No. 3, pp. 437-459.

Kar, S. and Marjit, S. (2001), "Informal sector in general equilibrium: welfare effects of trade policy reforms", *International Review of Economics and Finance*, Vol. 10 No. 3, pp. 289-300.

Karan, A.K. and Selvaraj, S. (2008), "Trends in wages and earnings in India: increasing wage differentials in a segmented labour market", International Labour Organization Asia-Pacific Working paper series. New Delhi.

Kumar, A. (2016), "Estimation of the size of the black economy in India, 1996–2012", *Economic and Political Weekly*, Vol. 51 No. 48, pp. 36-42.

Kumar, M. and Ranjan, R. (2015), "Wage differential between informal and formal wage worker in India", *Academic Journal of Economic Studies*, Vol. 1 No. 4, pp. 9-19.

La Porta, R. and Shleifer, A. (2008), "The unofficial economy and economic development", *Brookings Papers on Economic Activity, Fall*: pp. 275-352.

La Porta, R. and Shleifer, A. (2014), "Informality and development", *Journal of Economic Perspectives*, Vol. 28 No. 3, pp. 109-126.

Lingens, J. (2007), "Unions, wage setting, and economic growth", *Economic Modelling*, Vol. 24 No. 1, pp. 167-188.

Marjit, S. (2003), "Economic reform and informal wage – a general equilibrium analysis", *Journal of Development Economics*, Vol. 72 No. 1, pp. 371-378.

Marjit, S. and Acharya, R. (2003), *International Trade, Wage Inequality and the Developing Economies: A General Equilibrium Approach*, Physica-Verlag, New York, NY.

Marjit, S. and Beladi, H. (2002), "The stolper–samuelson theorem in a wage differential framework", *The Japanese Economic Review*, Vol. 53 No. 2, pp. 177-181.

Marjit, S. and Kar, S. (2009), "A contemporary perspective on the informal labour market: theory, policy and the Indian experience", *Economic and Political Weekly*, Vol. 44 No. 14, pp. 60-71.

Marjit, S. and Kar, S. (2011), *The Outsiders: Economic Reform and Informal Labour in a Developing Economy*, Oxford University Press India, New Delhi.

Marjit, S., Kar, S. and Acharyya, R. (2007), "Agricultural prospects and informal wage in general equilibrium", *Economic Modelling*, Vol. 24 No. 3, pp. 380-385.

Mazumdar, D. (1983), "Segmented labour markets in LDCs", *American Economic Review*, Vol. 73, pp. 254-259.

Mazumdar, D. (1993), "Labour markets and adjustment in open asian economies: the Republic of Korea and Malaysia", *The World Bank Economic Review*, Vol. 7 No. 3, pp. 349-380.

Narayanan, A. (2015), "Informal employment in India: voluntary choice or a result of labor market segmentation", *Indian Journal of Labour Economics*, Vol. 58 No. 1, pp. 119-167.

Neary, P. (1978), "Dynamic stability and the theory of Factor-Market distortions", *The American Economic Review*, Vol. 68 No. 4, pp. 671-682.

Santra, S. (2014), "Non-homothetic preferences: explaining unidirectional movements in wage differentials", *Journal of Development Economics*, Vol. 109, pp. 87-97.

Schneider, F. (2012), "The shadow economy and work in the shadow: What do We (not) know?", Discussion Paper No. 6423, IZA, Bonn, Germany.

Schneider, F., Chaudhuri, K. and Chatterjee, S. (2003), "The size and development of the Indian shadow economy and a comparison with other 18 asian countries: an empirical investigation", Department of Economics, Johannes Kepler University Linz, Austria, Economics working papers 2003-02.

Tansel, A. and Kan, E.O. (2012), "The formal/informal employment earnings gap: evidence from Turkey", Discussion Paper No. 6556. Institute for the Study of Labor, Bonn.

Turnham, D. (1993), "*Employment and Development: A New Review of Evidence*, " OECD, Paris.

Turnovsky, S.J. and Basher, M.A. (2009), "Fiscal policy and the structure of production in a two-sector developing economy", *Journal of Development Economics*, Vol. 88 No. 2, pp. 205-216.

Uzawa, H. (1961), "On a Two-Sector model of economic growth", *The Review of Economic Studies*, Vol. 29 No. 1, pp. 40-47.

Yabuuchi, S. and Chaudhuri, S. (2007), "International migration of labour and skilled–unskilled wage inequality in a developing economy", *Economic Modelling*, Vol. 24 No. 1, pp. 128-137.

Further reading

Cole, W.E. and Sanders, R.D. (1985), "Internal migration and urban employment in the third world", *American Economic Review*, Vol. 75, pp. 481-494.

Economic Survey various issues. Government of India, Ministry of Finance, New Delhi.

National Sample Survey, *Various Rounds*, Department of Statistics, Government of India, New Delhi

Appendix 1. The wage setting rule

Union's utility function is given by

$$U(w_Y, L_Y) = L_Y[u(w_Y) - u(w_X)], \tag{10}$$

where u follows the standard properties: $u' > 0$ and $u'' < 0$ and w_X is treated parametrically by the union.

The maximisation problem reads:

$$\underset{w_Y}{Max}\ U(w_Y, L_Y)$$

Subject to:

$$\frac{\partial}{\partial L_Y}\pi_Y(w_Y, L_Y; K_Y) = 0 \tag{10.1}$$

Further:

$$\frac{\partial}{\partial L_X}\pi_X(w_X, L_X; K_X) = 0 \tag{10.2}$$

$$L_Y + L_X = \overline{L} \tag{10.3}$$

The conditions given by equations (10.1) and (10.2) give the labour demand curves (MP_L) for the formal and informal sector, respectively. Equation (10.3) is nothing but the full employment condition. The sequence of variable determination is this. Tangency of the union's indifference curve and firm's labour demand curve determines w_Y and L_Y. Equation (10.3) determines L_X, and then equation (10.2) determines w_X.

Putting equation (10.1) into the utility function we get an unconstrained problem:

$$\underset{w_Y}{Max}\ U\left[w_Y, L_Y^d(w_Y; K_Y)\right].$$

The first order condition (FOC) is $\frac{dU}{dw_Y} = 0$: $\frac{\partial U}{\partial w_Y} + \frac{\partial U}{\partial L_Y}\frac{\partial L_Y}{\partial w_Y} = 0$

$$L_Y u'(w_Y) + \frac{\partial L_Y}{\partial w_Y}[u(w_Y) - u(w_X)] = 0$$

$$\frac{L_Y}{w_Y}\left[w_Y u'(w_Y) + \{u(w_Y) - u(w_X)\}\frac{w_Y}{L_Y}\frac{\partial L_Y}{\partial w_Y}\right] = 0$$

$$\frac{u(w_Y) - u(w_X)}{w_Y u'(w_Y)} = \frac{1}{\gamma_{LY}}, \tag{10.4}$$

where γ_{LY} is the elasticity of labour's marginal physical products and is constant for a CRS production function. As $\gamma_{LY} < 1$, there is a utility mark-up over the utility associated with the market clearing wage (under market clearing wage, $w_Y = w_X$ always). Note that w_Y and w_X are equilibrium values of wages, as they satisfy the FOC.

The FOC equation (10.4) can be used to find out the equilibrium comparative statics effect of the informal wage on the formal wage. Using the implicit function theorem:

$$\frac{dw_Y}{dw_X} = -\frac{u'(w_X)}{w_Y u''(w_Y)} > 0 \ \text{ as } u' > 0 \text{ and } u'' < 0$$

Hence, the equilibrium wage adjustment rule can be written in a reduced form as:

$$w_Y = \alpha(w_X) \text{ or equivalently } \hat{w}_Y = e_\alpha \hat{w}_X, \tag{12}$$

where $e_\alpha = \frac{w_X}{w_Y}\frac{dw_Y}{dw_X}$.

Further, $\frac{d^2 w_Y}{dw_X^2} = -\frac{u''(w_X)}{w_Y u''(w_Y)} + \frac{u'(w_X)\frac{dw_Y}{dw_X}\left[u''(w_Y) + w_Y u'''(w_Y)\right]}{\left[w_Y u''(w_Y)\right]^2}$.

We consider the class of concave utility functions for which the third derivate, u''', is either negligibly small $[u(w) = \sqrt{w}]$, or zero $\left[u(w) = aw - bw^2, w < \frac{a}{2b}\right]$, or negative $\left[u(w) = aw - bw^3, w < \sqrt{\frac{a}{3b}}\right]$. Under such condition, $|u''(w_Y)| > |w_Y u'''(w_Y)|$ so that overall, $\frac{d^2 w_Y}{dw_X^2} < 0$. Therefore, $w_Y = \alpha\ (w_X)$ is concave in w_Y-w_X plane, so that $0 < e_\alpha < 1$.

Intuitively, a rise in equilibrium w_X due to a rise in K_Y (Figure 2) is accompanied by a less-than-proportionate (equilibrium) increase in w_Y due to strict concavity of the union's utility function $U(w_Y, L_Y)$, that is, the indifference curves are strictly convex to the origin. It is optimal for the union to 'balance' out the effect over both the wage rate and employment. Contrast this with the situation under the market clearing wage (without a labour market distortion created by a formal-sector union) – a rise in K_Y (or K_X) causes an equal increase in both formal and informal wages.

Similar results can be obtained with a right to manage model that incorporates a Nash bargaining solution. It is well known that under Nash bargaining, the wage premium is less (the utility mark-up is smaller) compared to the monopoly model as the firm also bargains over profit – see Heijdra (2009; Ch 7, Section 7.1.2) for details.

Appendix 2. Behaviour of informal sector's output with investment

From equation (4), in proportionate terms,

$$\hat{K}_X = \hat{a}_{KX} + \hat{X}.$$

Using equation (20) and substituting for $\hat{w}_X$,

$$\hat{K}_X = \frac{\theta_Y \gamma_{LX} \theta_{LX}}{A\theta_Y + B\lambda_{LY}} \hat{K}_X + \hat{X}$$

$$\hat{X} = \left[1 - \frac{\theta_Y \gamma_{LX} \theta_{LX}}{A\theta_Y + B\lambda_{LY}}\right] \hat{K}_X.$$

Now $\hat{X} > 0$ if $A\theta_Y + B\lambda_{LY} - \theta_Y \gamma_{LX} \theta_{LX} > 0$
or if, $[A(1-\theta_X) - B\lambda_{LX}] + [B - \theta_Y \gamma_{LX} \theta_{LX}] > 0$
$[A(1-\theta_X) - B\lambda_{LX}] > 0$ as $\hat{p} > 0$.
Now, $B - \theta_Y \gamma_{LX} \theta_{LX}$

$$= \theta_X(1-\gamma_{LX}) + \theta_Y e_\alpha (1-\gamma_{LY}) + \gamma_{LX}\theta_{LX} - \theta_Y \gamma_{LX}\theta_{LX}$$
$$= \theta_X(1-\gamma_{LX}) + \theta_Y e_\alpha (1-\gamma_{LY}) + \gamma_{LX}\theta_{LX}(1-\theta_Y) > 0$$

Hence, $\hat{X} > 0$.

Corresponding author
Subhasankar Chattopadhyay can be contacted at: subhasankarc@iimidr.ac.in