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Multi-product maritime inventory routing with optional cargoes : An application to outbound automotive logistics

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An application to outbound automotive logistics

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Abstract

Purpose – The ocean transportation of automobiles is carried out by specialized Roll-on/Roll-off ships, which are designed to carry a large number of automobiles at a time. Many of these shipping companies have vertically integrated or collaborated with other logistics services providers to offer integrated maritime logistics solution to car manufacturers. The purpose of this study is to develop an optimization model to address the tactical level maritime logistics planning for such a company.

Design/methodology/approach – The problem is formulated as a mixed integer linear program and we propose an iterative combined Ant colony and linear programming-based solution technique for the same.

Findings – This paper can integrate the maritime transportation planning of internally managed cargoes with the inventory management at the loading and discharging ports to minimize supply-chain cost and also maximize additional revenue through optional cargoes using same fleet of ships.

Research limitations/implications – The mathematical model does not consider the variability in production and consumption of products across various locations, travel times between different nodes, etc.

Practical implications – The suggested mathematical model to the supply-chain planning problem and solution technique can be considered in the development of decision support system for operations planning.

Originality/value – This paper extends the maritime inventory routing model by considering simultaneous planning of optional cargoes with internally managed cargoes.

Keywords Maritime logistics, Inventory routing, Optional cargoes, Outbound automotive logistics, Ant colony algorithm, Inventory management, Optimization techniques

Paper type Research paper

1. Introduction

The automobile industry's supply chain broadly consists of two parts: the first involving supply of raw materials and components to various manufacturers till the final assembly stage; and the second part involving the delivery of finished automobiles from the factory to the dealers (or final customers). The later part can be referred to as outbound automotive logistics. We focus on the maritime outbound logistics of automobiles. With globalization of business especially the automotive trade, manufacturing activities are moving to developing regions like India, China, Brazil, etc. from the traditional blocks like USA, Europe, etc. This change is motivated by cost reduction as well as proximity to emerging markets. This decentralization of automotive trade has led to new logistics challenges for the industry as a whole.

Japan and South Korea are major automobile exporters and most of the automobiles shipping routes originate there. USA is the largest importer of automobiles followed by continental Europe. The export of vehicles from countries like India, China, etc. is

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rising at a very fast rate, although the present share is less. The import demand is also on a rise from new emerging markets like Russia, Brazil, India and China. According to a Market Data report published by Mitsui O.S.K. Lines Ltd. in 2013, the average monthly export volume of new vehicles from Japan stood at around 3,80,000 units from 2010 to 2012 (MOL, 2013). The deep-sea shipping is the most important means of transport for this type of cargo.

The use of maritime transportation for finished automobile trade across the globe has led to the realization of economies of scale in transportation. Car carriers or Roll-on/Roll-off ships are sea going vessels designed to carry wheeled cargo such as automobiles, trucks, trailers, etc. that are driven on and off the ships on their own wheels. The carrying capacity of these ships is measured in terms of car equivalent units (CEUs). The carrying capacity of modern day car carriers ranges from around 1,000 to 6,500 cars on a single voyage. A few large operators offer Roll-on/Roll-off capacity for the transportation of such cargo. These companies operate 30-60 vessels of such type. Each of the ship may vary in characteristics like size, speed, capacity, running cost, etc. The operators negotiate contracts directly with the car manufacturers. Most of the contracts are long-term contracts with time horizons ranging from two to five years. In some of the trades, operators co-operate in order to offer a higher frequency and thereby a better service (Kroneberg and Ramberg, 2001).

The Roll-on/Roll-off operators are facing serious threat from container shipping business. They must improve their profitability and enhance customer service to retain their position as a dominant mode of transportation in this sector (Øvstebø *et al.*, 2011a). Most of the major Roll-on/Roll-off shipping lines are emerging as third-party logistics providers, either individually or in collaboration with other large logistics services providers. These companies are offering complete logistics solutions to the auto manufacturers for their international maritime logistics. In these types of contracts between auto manufacturers and the mega-carriers the inventory management at the ports, inland transportation as well as ocean transportation falls at the responsibility of the shipping line. We use the term internal cargo for this type of contract. The shipping lines also carry long-term contractual cargoes, with only maritime transportation at their responsibility. We use the term external mandatory cargo for these types of contracts. In addition they also carry optional cargoes, referred to as external optional cargoes, from small players or from other carriers at specified revenue per unit of cargo, if profitable.

A Roll-on/Roll-off ship is classed under general cargo ships as per UNCTAD (2012). This type of ship varies in characteristics from other types of ships. The Roll-on/Roll-off ships run in liner mode as they publish schedules in advance, although they do not follow closed loops as the container ships and frequently change routes as per requirements. In a single voyage a Roll-on/Roll-off ship may not necessarily return to the origin port as a container ship would do. In the short term a Roll-on/Roll-off shipping line has to service a given set of ports along a given geographical route using a fleet of vessels. The decision to optimally route the ships along the given ports may follow the expected cargo information. The shipping scenario in which the shipping company manages the cargo transportation as well as inventory management at the ports can be modeled as an inventory routing problem (Brønmo *et al.*, 2007a). Thus although the Roll-on/Roll-off ships worldwide run in liner mode, in the short term this type of shipping can be modeled as a combination of inventory routing problem with routing of contractual and optional cargoes. This model can be used tactical level for the determination of optimal fleet routing.

We consider a short-term planning problem for a given number of ships engaged in pick-up and delivery of cargo along a given set of geographical regions. The problem is defined from the view of a shipping company operating a fleet of Roll-on/Roll-off ships, which offers integrated maritime logistics services to some auto-manufacturers apart from usual liner services. The planning problem involves multiple ships and includes the simultaneous planning of inventory constrained routing of some contractual cargo along with only cargo routing of other mandatory and optional cargoes. This model is also applicable to scenarios in industrial and tramp shipping in which a part of transportation demand is in form of inventory routing and remaining cargoes require only transportation planning. We develop a mixed integer non-linear programming (MINLP) model for the problem with suitable linearization techniques. We propose a two-step iterative solution technique for the model, in which an ant colony optimization (ACO)-based algorithm, determines the ship routes followed by a linear programming-based solver to determine corresponding schedules and shipment plans.

In the next section we discuss the related literature. Section 3 gives a detailed description of problem under study. In the Section 4 we present the mathematical formulation for the problem. Section 5 presents the solution method proposed. Experimental setup and findings are discussed in Section 6. We conclude our work with Section 7.

2. Related literature

The application of operations research techniques as planning aid is widely studied in the context of maritime transportation. This is evident from the steady rise in the research interest in this field. Ronen (1983, 1993), Christiansen *et al.* (2004, 2013) and Kjeldsen (2011) can be cited as some important review works in this area. The number of references has increased from 40 in the first review by Ronen (1983) to more than 100 in the latest review published by Christiansen *et al.* (2013). The given problem context is closely related to problems corresponding to maritime supply-chain planning and Roll-on/Roll-off ship planning. We will discuss literature related to these topics.

Research on maritime inventory routing problem has been mainly studied in the context of industrial shipping. Al-Khayyal and Hwang (2007) classify the maritime routing and scheduling problems into inventory routing and cargo routing problems, depending on whether the problem deals with integrated supply-chain planning or only transportation planning, respectively. Christiansen (1999) presents an inventory constrained ship routing and scheduling problem for a single commodity faced by an ammonia producer. Each production and consumption port has a fixed production/demand rates for the product and a specified minimum and maximum stock capacity for the product. The company manages a fixed fleet of heterogeneous vessels for ammonia transportation. A time-continuous mathematical formulation is presented for the problem, in which each port is assigned a certain number of arrivals in planning horizon. The papers by Christiansen and Nygreen (1998a) and Christiansen and Nygreen (1998b) compliment this research work. A mixed integer linear programming formulation is presented for the problem and a DW decomposition-based solution approach suggested. Flatberg *et al.* (2001) suggest an iterative improvement heuristic combined with an LP solver as a solution scheme for the same problem. Ronen (2002) addresses a multi-product maritime inventory routing problem of a similar structure. A two-stage solution method is suggested: first, a shipment plan is generated with quantities for loading/discharging, time windows for arrival, etc. and then a ship routing and scheduling problem can be solved using existing techniques. An MIP formulation is presented and a cost-based heuristic suggested as solution technique for the shipment-planning problem. Al-Khayyal and Hwang (2007) extend the model presented by Christiansen (1999) to address the combined inventory constrained maritime routing and scheduling of multiple commodities in form of liquid bulk. The context is of a chemical transportation company with a fixed fleet of heterogeneous ships, involved in the pick-up and delivery of multiple oil products across an archipelago of islands. The products are assumed to require dedicated compartments in the ships. Each port has a separate storage capacity for the products it produces or consumes, such that inventory level for each product has to be maintained between minimum and maximum pre-specified levels. The products associated with a port are assumed to have a constant rate of demand or production for the planning horizon. An MINLP model is proposed for the problem and it is reformulated as a mixed integer linear program. Multiple test cases are generated and solved using a commercial solver. Some specific maritime logistics applications as extensions of maritime inventory routing problem are presented in the context of cement transportation (Christiansen *et al.*, 2011), liquefied natural gas (LNG) transportation (Gronhaug and Christiansen, 2009; Grønhaug *et al.*, 2010; Andersson *et al.*, 2010; Halvorsen-Weare and Fagerholt, 2013), vacuum gas oil (VGO) transportation (Furman *et al.*, 2011) and crude oil transportation problem (Shen *et al.*, 2011).

Persson and Gothe-Lundgren (2005) extend the maritime inventory routing problem to include process schedule at the plants and consider variable production rates. Bredström *et al.* (2005) present an extension of multi-product maritime inventory routing problem to address wood-pulp distribution problem. Song and Furman (2013) try to include practical considerations into a single product maritime inventory routing problem and present a time discrete network-based formulation for the problem. Dauzere-Peres *et al.* (2007) present the case of successful implementation of optimization-based Decision Support System (DSS) for supply-chain planning of a raw material, calcium carbonate slurry to European paper manufacturers. The inventory routing problem discussed is a multi-commodity inventory routing problem having a single origin and multiple destinations with time-varying demands across the planning horizon. Agra *et al.* (2013) address the short-term short-sea fuel oil distribution problem for an archipelago. The problem considered in this paper is a multi-product, finite-horizon IRP with deterministic but time-varying demand. Inventory considerations are applicable only at the demand side. Multiple time windows are considered for each harbor arrival. The mathematical model is formulated including a combination of continuous and discrete time horizon. The paper discusses several strategies to improve the proposed model, such as tightening bounds, using extended formulations and including valid inequalities.

The Roll-on/Roll-off shipping has received much lesser attention in maritime transportation literature as compared to other modes. This may be due to small share of these types of ships in world fleet and also due to their similarity to general cargo ships. Øvstebø *et al.* (2011a) have addressed an operational level problem related to the optimum stowage planning for Roll-on/Roll-off ships. Once the ship routes are fixed, the decisions include which cargoes to carry, how much vehicles to carry from each cargo, and how to stow the vehicles carried during the voyage. An MILP model is developed for the problem, a heuristic proposed as solution approach and computational results shown as comparison. Øvstebø *et al.* (2011b) extend the problem to include stowage planning considerations into routing and scheduling decisions at the operational level. A MILP formulation is presented for the Roll-on/Roll-off ship routing and stowage plan problem. As solution approaches, a standard MILP solver is compared with a multi-step heuristic algorithm.

Some important gaps can be cited from our literature review. The problems related to inventory maritime routing do not consider the combined planning of optional cargoes for additional revenue generation. The literature on Roll-on/Roll-off ships considers some problems related to routing and stability. The fact that the companies operating or managing these types of carriers provide complete end-to-end maritime logistics services to some automobile exporters can be considered as a opportunity to model the supply-chain planning problem as an extension to the existing inventory maritime routing problem discussed in the extant literature.

3. Problem description

The planner has a fixed number of ships available for a given time interval, in which the ship should pick-up and deliver cargoes. A cargo implies a set of vehicles from a single car-manufacturer, usually of same make and type, originating from a particular port in one geographical region and destined for a different port in another geographical region. There are three types of cargoes to be serviced by this fleet of vessels: internally managed cargoes, externally managed cargoes and optional cargoes. For the first set of cargoes there is no explicit time windows, as they are derived from inventory constraints at company managed automobile terminals. There are time-windows associated with each pair of pickup and delivery ports in case of second and third types of cargoes. In this formulation we keep the number of units of cargoes for mandatory external cargoes to be a fixed agreed upon number. The consignments of optional cargoes also consist of a fixed number of units of cargoes with given total revenue for transportation service. We do not consider the effects of cargo stowage onboard, as this would complicate the formulation for multiple ships.

The planner is responsible for choosing the sequence of ports to be visited by each ship in the planning horizon. The planner needs to decide which cargoes to be serviced by which ship and which of the optional cargoes to be accepted in the given routing schedule. In case of an internally managed cargo, the planner needs to decide the shipment sizes, number of arrivals at each origin and destination ports along with time of start of service at each arrival and include these arrivals in overall ship routes. The overall objective is to minimize the transportation costs for ships and inventory costs at self managed terminals along with maximizing revenue from the optional cargoes.

4. Model formulation

In this section we describe a mathematical model for the problem under consideration. The model is developed along the lines of Al-Khayyal and Hwang (2007) and Øvstebø *et al.* (2011b), but with significant modifications to account the integrated planning of internally managed cargoes with external mandatory and optional cargoes and some features of the problem specific to Roll-on/Roll-off shipping.

A node is defined as a harbor and arrival number to that harbor. For the internally managed ports the arrival number can be more than one and for externally managed terminals the number of arrivals is fixed to be one. This type of nodal structure is followed to integrate the cargo routing of external cargoes into an inventory maritime routing model. As opposed to the inventory maritime models discussed so far in literature, the stock for holding cargo in case of vehicle terminals is common for all cargo sets rather than a separate stock for each cargo type. Also the carrying space on ships is common and continuous for all cargoes. To keep the notations simple we assume that all the ships are in ports at the star of the planning horizon. Although it is easily possible to incorporate ships starting position as a point at sea by introducing dummy harbors at the exact location with no cargoes to be loaded or discharged. We assume that two ships cannot simultaneously load or discharge at a particular harbor and there are no setup times and costs involved between sequentially loading or discharging two different sets of cargoes at the same port. We have followed the same notations used by Al-Khayyal and Hwang (2007) for the ease of understanding. To illustrate the problem configuration we present a small test problem as illustration in Figure 1. This problem instance consists of two ships 1 and 2, three internal harbors 1, 2, and 3, two internal cargoes, a pair of origin-destination harbors for external mandatory cargo 3 and a pair of origin-destination harbors for external optional cargo 4. Two positions each are defined for all three internal harbors. The ships 1 and 2 are initially located at harbors 1 and 3, respectively. For the case of this test problem, the planning problem is to determine the optimal routes of both the ships starting from their initial positions. The solid arrows in the diagram show the feasible paths from node (1, 2) and the dashed arrows show the feasible paths from node (1, 1). In an optimal solution to this planning problem, the ship 1 goes to position (3, 2) and then goes to pick up the external mandatory cargo at harbor pair 4-5. The ship 2 visits the internal position (2, 1) and then goes to serve the optional cargo at harbor pair 6-7.

In this paper we consider a formulation based on continuous time horizon. In this formulation, the decision variables are written in lowercase letters and the parameters and sets are written in uppercase letters.

Indices

i,j index for a harbor
m,n indices for arrival numbers at a harbor
v index for a ship
k index for a cargo set
i_v initial harbor of a ship v
m_v initial harbor arrival number of a ship v

Sets

V set of ships
S set of all harbor-arrival nodes
S_T set of all nodes corresponding to internally managed terminals
S_O set of initial nodes for all ships
S_N set on all non-initial nodes for all ships (S/S_O)
S_M set of non-initial nodes for all ships at internal harbors (S_T/S_0)

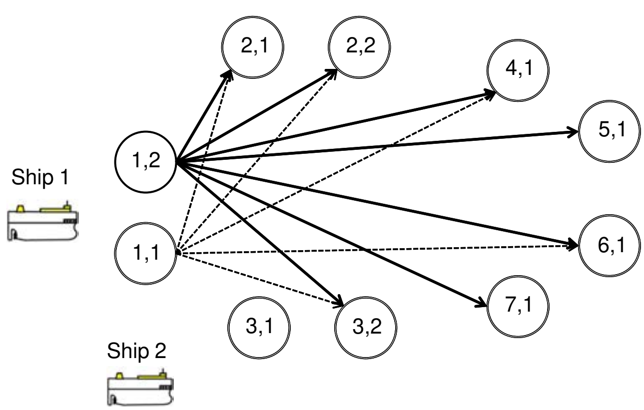

Figure 1.
Problem instance for illustration

S_N^v set of all possible visit nodes for ship v ($S_N \cup \{(i_v, m_v)\}$)
S_{os} set of nodes corresponding to optional cargoes
S_{cs} set of nodes corresponding to mandatory external cargoes
K set of all cargoes
K_t set of all internally managed cargoes
K_i set of cargoes handled at harbor i
K_{di} set of cargoes consumed by harbor i
K_{pi} set of cargoes produced by harbor i
K_v set of cargoes that can be loaded on ship v
H_t set of internally managed harbor terminals
H_{os} set of harbors associated with optional cargoes
H_{cs} set of cargoes associates with mandatory external cargoes
H set of all harbors ($H_t \cup H_{cs} \cup H_{os}$)

Parameters
T length of the planning horizon in days
C_{ijv} average sailing cost of ship v along the arc joining ports i and j
T_{ijv} average sailing time taken by ship v along the arc joining ports i and j
Q_{vk} quantity of product k on the ship v at the start of planning horizon
$CAPv$ aggregate capacity of ship v measured in terms of numbers of CEUs of vehicles
J_{ik} −1/0/1, depending upon whether harbor i consumes/doesn't handle/produces product k
R_{ik} daily production or consumption rate of product k at internal harbor i
Cw_{ik} per unit loading/unloading cost of product k at harbor i
TQ_{ik} per unit loading/unloading time taken for product k at harbor i
IS_{ik} initial stock of product k at internal harbor i

$S_{\min_i}$ minimum total stock-level at internal harbor i
$S_{\max_i}$ maximum total stock-level at internal harbor i

TW_{si} start of time window for start of service at an external harbor i
TW_{ei} end of time window for start of service at an external harbor i
Rev_i total revenue for serving a harbor i associated with an optional cargo
$RevI_{ik}$ per unit revenue for delivering internal cargoes
RT_v potential benefit per unit time for ending the route early for ship v
Q_i quantity to be serviced for an optional cargo harbor i
Ncs number of contractual external harbors
Nos number of optional external harbors

$M_{\min_i}$ minimum number of ship visits required at harbor i in the given planning horizon for a feasible shipment plan
$M_{\max_i}$ maximum number of ship visits defined at harbor i in the given planning horizon;

We sequentially number all the harbors starting from internally managed harbors, followed by contractual external harbors and finally optional external harbors. For external harbors each origin is associated with a destination harbor such that if number N_i is associated with origin harbor then the number $[N_i + (N_{os}$ or $N_{cs})/2]$ designates the destination harbor. Figure 2 illustrates the numbering scheme for all the harbors. N_{cs0} denotes the harbor number from which contractual external cargoes start

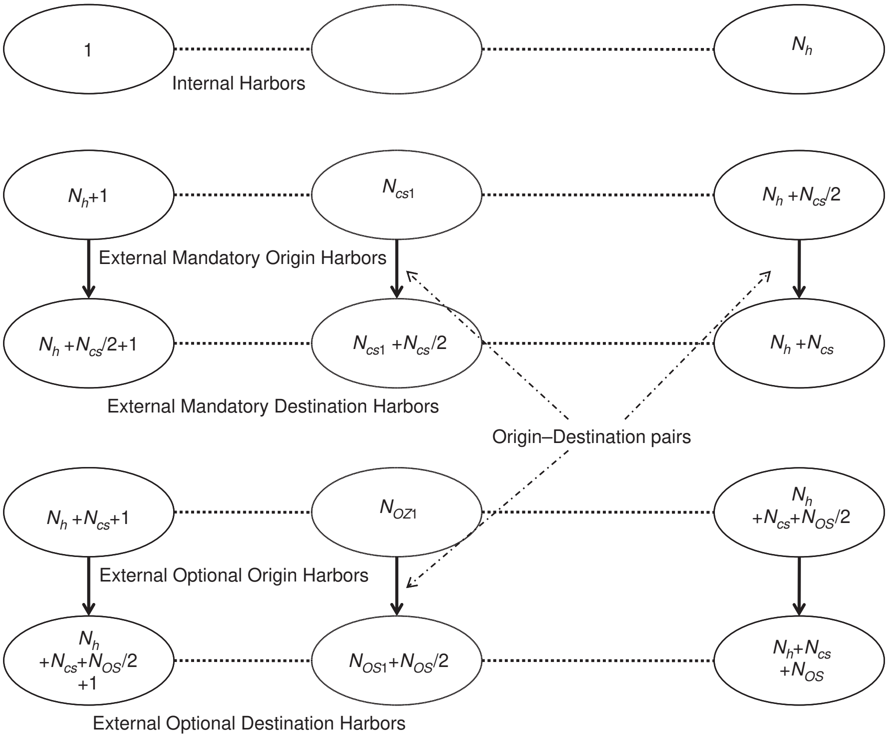

Figure 2. Numbering scheme for harbors

and N_{os0} denotes the harbor number from which optional external cargoes start. From the Figure 2 it can be seen that $N_{cs0} = N_h + 1$ *and* $N_{os0} = N_h + N_{cs} + 1$.

Decision variables

- x_{imjnv} 1 if ship v sails directly from node (i,m) to node (j,n), 0 otherwise
- z_{imv} 1 if ship v ends its route at node (i,m), 0 otherwise
- y_{im} 1 if node (i,m) remains unvisited by any ship in the planning horizon, 0 otherwise
- l_{imvk} Load onboard of product k on ship v just before leaving node (i,m)
- q_{imvk} Quantity of product k loaded/discharged at node (i,m) by ship v
- t_{im} Starting service time at node (i,m)
- t_{Eim} Ending service time at node (i,m)
- t_v Route ending time for ship v
- S_{imk} Stock level of product k at harbor i at the start of service at arrival m
- S_{Eimk} Stock level of product k at harbor i at the end of service at arrival m

Minimize:

$$\sum_{v\in V}\sum_{(i,m)\in S_N^v}\sum_{(j,n)\in S_N} C_{ijv}x_{imjnv} + \sum_{(i,m)\in S}\sum_{v\in V}\sum_{k\in K_v} Cw_{ik}q_{imvk} + \sum_{(i,m)\in S_{os}} Rev_i y_{im} - \sum_{(i,m)\in S_T}\sum_{k\in Kd_i}\sum_{v\in V} RevI_{ik}q_{imvk} - \sum_{v\in V} RT_v(T - t_v) \quad (1)$$

Subject to:

$$\sum_{(j,n)\in S_N} x_{i_v m_v jnv} + z_{i_v m_v v} = 1, \quad \forall v \in V \tag{2}$$

$$\sum_{(j,n)\in S_N^v; i\neq j} x_{jnimv} - \sum_{(j,n)\in S_N; i\neq j} x_{injmv} - z_{imv} = 0, \quad \forall (i,m) \in S_N, v \in V \tag{3}$$

$$\sum_{(i,m)\in S_N^v} z_{imv} = 1, \quad \forall v \in V \tag{4}$$

$$\sum_{v\in V} \sum_{(j,n)\in S_N^v} x_{jnimv} + y_{im} = 1, \quad \forall (i,m) \in S_N \tag{5}$$

$$y_{im} = 0, \quad \forall (i,m) \in S_{cs} \cup S_O \tag{6}$$

$$y_{i1} = y_{(i+N_{os}/2)1}, \quad \forall i \in H_{os} : i \leqslant (N_{os0} + N_{os}/2 - 1) \tag{7}$$

$$y_{im} - y_{i(m-1)} \geqslant 0, \quad \forall (i,m) \in S_M : m > M\min{}_i \tag{8}$$

$$x_{imjnv}(l_{imvk} + J_{jk} q_{jnvk} - l_{jmvk}) = 0, \quad \forall v \in V, k \in K_v, (i,m) \in S_N^v, \\ (j,n) \in S_N : i \neq j \tag{9}$$

$$q_{imvk} = \left(\sum_{(j,n)\in S_N^v : i\neq j} x_{jnimv} \right) Q_i, \quad \forall (i,m) \in S_{cs} \cup S_{os}, v \in V, \\ k \in K_i \cap K_v \tag{10}$$

$$Q_{vk} + J_{i_v k} q_{i_v m_v k} - l_{i_v m_v k} = 0, \quad \forall v \in V, k \in K_v \tag{11}$$

$$\sum_{k\in K_v} l_{imvk} \leqslant CAP_v, \quad \forall v \in V, (i,m) \in S \tag{12}$$

$$q_{imvk} \leqslant CAP_v \sum_{(j,n)\in S_N^v} x_{jnimv}, \quad \forall v \in V, (i,m) \in S_N, k \in K_v \tag{13}$$

$$t_{im} - t_{i(m-1)} \geqslant 0, \quad \forall (i,m) \in S_M : m > M\min{}_i \tag{14}$$

$$t_{im} + \sum_{v\in V} \sum_{k\in K_v} TQ_{ik} q_{imvk} - t_{Eim} = 0, \quad \forall (i,m) \in S \tag{15}$$

$$x_{imjnv}(t_{Eim} + T_{ijv} - t_{jn}) \leqslant 0, \quad \forall v \in V, (i,m) \in S_N^v, (j,n) \in S_N : i \neq j \tag{16}$$

$$y_{im}(t_{im} - t_{Ei(m-1)}) = 0, \quad \forall (i,m) \in S_M : m > M\min{}_i \tag{17}$$

$$t_{im}, t_{Eim} \leqslant T, \quad \forall (i,m) \in S \tag{18}$$

$$TW_{si} \leqslant t_{i1} \leqslant TW_{ei}, \quad \forall i \in H_{cs} \cup H_{os} \tag{19}$$

$$z_{imv}(t_v - t_{Eim}) = 0, \quad \forall (i,m) \in S, v \in V \tag{20}$$

$$s_{i1k} = IS_{ik} + J_{ik}R_{ik}t_{i1}, \quad \forall i \in H_t, k \in K_t \tag{21}$$

$$s_{imk} - \sum_{v \in V} J_{ik}q_{imvk} + J_{ik}R_{ik}(t_{Eim} - t_{im}) - s_{Eimk} = 0, \quad \forall (i,m) \in S_T, \quad k \in K_i \tag{22}$$

$$s_{Ei(m-1)k} + J_{ik}R_{ik}(t_{im} - t_{Ei(m-1)}) - s_{imk} = 0, \quad \forall (i,m) \in S_M, \quad k \in K_i : m > M\min_i \tag{23}$$

$$t_{im} - t_{Ei(m-1)} \geqslant 0, \quad \forall (i,m) \in S_M, k \in K_i : m > M\min_i \tag{24}$$

$$S\min_i \leqslant \sum_{k \in K_i} s_{imk} \leqslant S\max_i, \quad \forall (i,m) \in S_T \tag{25}$$

$$S\min_i \leqslant \sum_{k \in K_i} s_{Eimk} \leqslant S\max_i, \quad \forall (i,m) \in S_T \tag{26}$$

$$S_{Eimk} + J_{ik}R_{ik}(T - t_{Eim}) \geqslant 0, \quad \forall (i,m) \in S_T, k \in K_i m : M\max_i \tag{27}$$

$$\sum_{k \in K_i} (s_{Eimk} + J_{ik}R_{ik}(T - t_{Eim})) \leqslant S\max_i, \quad \forall (i,m) \in S_T, \quad k \in K_i : m = M\max_i \tag{28}$$

$$l_{imvk}, q_{imvk}, t_{im}, t_{Eim}, t_v, s_{imk}, s_{Eimk} \geqslant 0, \quad \forall (i,m) \in S, v \in V, k \in K \tag{29}$$

$$x_{imjnv}, y_{im}, z_{imv} \in \{0,1\}, \quad \forall (i,m),(j,n) \in S, v \in V \tag{30}$$

Objective expression (1) aims at minimizing the ship sailing costs and port costs comprising of cargo loading/unloading, while trying to maximize additional revenues from optional cargoes. The objective function also tries to maximize deliveries of internal cargoes by maximizing the revenue associated with their delivery. As opposed to other maritime transportation scenarios, here we assume no set-up cost or time associated with changing over between different cargoes, while loading/discharging. This makes sense in automobile terminals, as cargo sets are arranged in advance for driving into/off the Roll-on/Roll-off vessels, sequentially. Although the inventory management of internal cargoes lies with the planner, still she would like to maximize the shipments in a given planning horizon, so we would associate a per unit profit with the internal cargoes and include it in the objective term. This is a surrogate revenue term meant to avoid building up of inventory on the ships and origin harbors at the end of planning horizon. It would be beneficial to finish the ship routes early so that they are ready to pick up newer cargoes. This is also a surrogate revenue term and aims at finishing the individual ship routes early for maximizing future business potential.

Constraints (2) are the initial position constraints, which ensure that each ship must depart from its initial position or not move at all and stay at the initial position throughout the planning period. Constraints (3)-(8) are ship routing constraints. Constraints (3) are flow conservation constraints which ensure that the mth arrival to harbor i should leave that harbor to go to another harbor, or end its route there. Constraints (4) are route-finishing constraints that ensure that each ship must end its route somewhere. Constraints (5) are one-time visit constraints, which ensure that each harbor arrival is visited at most once. A harbor arrival may not be visited at all; in that case y_{im} is equal to 0. Constraints (6) ensure that initial ship positions and nodes corresponding to contractual harbors are compulsorily visited during the planning horizon. Constraints (7) ensure that if a ship visits origin node corresponding to an optional cargo, then it must visit its associated destination node. Constraints (8) are arrival sequence constraints, which ensure that if an arrival number to a harbor remains unvisited by any ship then all the higher numbered arrivals to that harbor must remain unvisited.

Constraints (9)-(13) are related to loading/discharging of cargo. Constraints (9) are ship loading constraints which ensure that the final load of a cargo k onboard a ship v at the end of arrival node (j,n) is equal to the load of cargo k onboard the ship at last node, plus (or minus) the quantity loaded (or discharged) on to the ship at this node. The whole term is multiplied with the flow binary variable, which ensures that this constraint holds only in case the node (j,n) is immediately followed by the node (i,m) in the route of ship v. This is a non-linear expression and can be linearized by replacing the non-linear expression with a set of two constraints as shown below (Williams, 1978). The term CAP_v is an upper bound on the expression within the parenthesis $(l_{imvk}+J_{ik}q_{jnvk}-l_{jnvk})$:

$$l_{imvk}+J_{ik}q_{jnvk}-l_{jnvk}+CAP_v x_{imjnv}\leqslant CAP_v, \quad \forall v\in V, k\in K_v, \\ (i,m)\in S_N^v, (j,n)\in S_N \tag{31}$$

$$l_{imvk}+J_{ik}q_{jnvk}-l_{jnvk}-CAP_v x_{imjnv}\geqslant -CAP_v, \quad \forall v\in V, k\in K_v, \\ (i,m)\in S_N^v, (j,n)\in S_N \tag{32}$$

Constraints (10) are external cargo load constraints and fix the quantity loaded or discharged from ship v of product k at an external harbor i to be equal to the pre-specified number associated with that cargo if the ship v visits that harbor, 0 otherwise. Constraints (11) are initial load constraints which ensure that the final load of a product on a ship at the departure from its starting node must be equal to the initial load of the product on that ship plus or minus the quantity of that cargo loaded or discharged at the ships starting node. Constraints (12) are ship capacity constraints and ensure that the total load onboard the ship at any time must not exceed the ship's maximum capacity. Constraints (13) are quantity feasibility constraints, which ensure that the quantity loaded/discharged on a ship of a cargo at a node is >0 only when the ship visits that node.

Constraints (14)-(20) are related to arrival/departure timings at the nodes. Constraints (14) are service time sequence constraints that ensure that the starting service time at an arrival to a harbor is greater than or equal to the service starting time of the previous arrival. Constraints (15) give the relationship between the end of service time and start of service time at a node. The time is spent in loading/discharging of cargoes. We do not consider set-up timings between successive (un)loadings of different cargo sets, as these are similar cargoes and arranged in advance for sequential loading or discharging. Constraints (16) are route-schedule compatibility constraints and ensure that the arrival time at a node is greater than or equal to the departure time at the immediate predecessor node plus the traveling time between the two nodes. This is a non-linear constraint and the product with the binary variable x_{imjnv} ensures that the relationship holds only when the two nodes are directly connected along a ships route. These non-linear constraints can be easily linearized in the same way as Constraints (9), by replacing the expression with a set of two linear constraints shown below:

$$t_{Eim} + T_{ijv} - t_{jn} + T.x_{imjnv} \leqslant T, \quad \forall v \in V, (i,m) \in S_N^v, (j,n) \in S_N : i \neq j \tag{33}$$

$$t_{Eim} + T_{ijv} - t_{jn} + T.x_{imjnv} \geqslant -T, \quad \forall v \in V, (i,m) \in S_N^v, (j,n) \in S_N : i \neq j \tag{34}$$

We have assigned non-zero times to non-visited nodes. According to Constraints (18), all starting and ending times associated with a non-visited arrival to a harbor is equal to the ending time of the preceding arrival to that harbor. This is also a non-linear constraint and can be linearized in the same way as Constraints (9) and (16) as shown earlier. Constraints (18) set the upper bounds on time variables to be within the planning horizon, T. Constraints (19) ensure that the timing of arrival at the external harbors lies within the agreed upon time windows for start of service. Constraints (20) assign values to the route finishing time for each ship. When the ship v ends its route z_{imv} is equal to 1, then t_v is equated to finishing time at that last arrival. This is a non-linear constraint and can be linearized as shown below in Constraints (35) and (36). Here T is the upper bound on the value inside the parentheses ($t_v - t_{Eim}$):

$$t_v - t_{Eim} + z_{imv} T \leqslant T, \quad \forall (i,m) \in S, v \in V \tag{35}$$

$$t_v - t_{Eim} + z_{imv} T \geqslant -T, \quad \forall (i,m) \in S, v \in V \tag{36}$$

Constraints (21)-(29) are related to stock level management at the internal harbors. Constraints (21) are initial inventory constraints and ensure that stock level of product at first arrival to a harbor is equal to its initial inventory at the harbor plus (minus) the quantity produced (consumed) at the harbor till the starting service time. For the harbors where ships are initially positioned, the initial stock would be equal to the initial inventory of the product. Constraints (22) give the relationship between the end stock level of a product at a harbor and its starting stock level in a particular arrival. The difference is stock levels arise due to the production or consumption taking place in this time and any ship loading or discharging the cargo at this arrival. Constraints (23) give the relationship between the starting stock level of a product at an arrival to a harbor and its end stock level at the preceding arrival to that harbor. This constraint does not allow more than one ships to berth at the same time in a harbor. We have taken this assumption considering that the time spent at ports is relatively small compared to sailing times.

Constraints (25) and (26) ensure that the total stock of all products at a harbor should be within the maximum and minimum bounds. Constraint (27) ensures that the final stock of a product at a harbor at the end of planning horizon should not be negative, i.e. there should not be a backlog in demand. Constraint (28) ensures that the total stock of all the products at a harbor at the end of the planning horizon should not exceed the maximum limit. Constraints (29) ensure positive values for all continuous variables and Constraints (30) enforce binary values for the binary variables.

5. Solution method

We refer to the solution techniques used by Flatberg *et al.* (2001) to solve a single product maritime inventory routing problem and Bredström *et al.* (2005) for a distribution problem. The solution to the problem of scheduling the vessels will consist of a set of calls to the internal harbors, and single calls to the corresponding origins and destinations of each external and selected optional cargo, after determining which ones to carry. A solution to a problem instance has to satisfy two types of feasibilities. The first part is represented by Constraint sets (2)-(8). These constraints are related to the routes taken by the vessels only and take care of the combinatorial feasibility part of the problem. The remaining sets of constraints from (9) to (28) are related to the quantities of loading/discharging and arrival times at harbors and define the linear feasibility part of the problem.

As a solution approach we propose an iterative algorithm that works in three stages: an ACO based to generate a ship routing; a LP solver to check the feasibility of the generated route and obtain corresponding optimal shipment plan and ship schedules; and a local search step to improve the existing ship routing. The fitness value of a feasible route generated in an iteration of the algorithm is given as a sum of travel cost of the route and other costs derived from the objective value of corresponding LP. The fitness value associated with a feasible plan is used as information to update some parameters of the ACO algorithm, which guide it to find improved solutions in further iterations. The ACO-based heuristic technique offers certain advantages over local search-based metaheuristic techniques. First we can incorporate all the combinatorial constraints in the route building stage itself. That means all the routes that we get as a solution from our ACO heuristic satisfy combinatorial feasibility. As shown later we can also avoid some time and quantity infeasible routes right at the route building stage. Second we do not have to start with an initial solver. ACO being a random search technique is expected to exploit the solution space well.

5.1 ACO algorithm for the generation of ship routing plans

ACO is a probabilistic technique for computational problems that can be reduced in the interests of determining optimal paths via graphical analysis. There are several versions of ACO algorithm. One of the most successful versions is Ant colony systems (ACS) algorithm. We use a modified version of ACS in our solution algorithm.

5.1.1 Graph creation for ACS. A node in the graph is represented as a triplet of (v, i, m), where v stands for a ship, i stands for a harbor and m stands for arrival number to that harbor. Thus a node in the graph represents a feasible arrival of a particular ship to a harbor at a particular arrival number. This arrival of ship at a particular harbor position can be defined as an event in the entire ship routing operation. We also assign two dummy nodes as an origin node and an end node to the graph. The directed arc joining two nodes represents that the start timing of the event joining the arrowhead is either greater than or equal to the node on the tail or completely unrelated to the event on the tail. Any node lying on a path after a particular node follows the same relationship with the node visited earlier. This type of nodal structure makes it easier to follow precedence relationships among ship visits to port arrivals, both from the point of view of ship routes and from the port arrival sequence perspective.

5.1.2 Definitions and notations. In this section we define variables and parameters related to the ACS algorithm, based on the graphical structure defined in earlier section. We will also use some notations defined earlier in Section 4.

a,b nodes in the network graph, where $a \equiv (i,m,v)$ and $b \equiv (j,n,u)$, $\forall (i,m),(j,n) \in S$; $v,u \in V$

N set of all the nodes in the network graph

N_v set of all nodes corresponding to ship v

num_itrs	parameter which determines the number of iterations the algorithm will run
num_ants	parameter which determines the number of ants used in a single iteration
ENode	An artificially assigned end node
α, β_R, β_C, ξ	parameters used in the ACO algorithm
ρ, q_0, τ_0, C_0	parameters used in the ACO algorithm
NodeSet	set of non-visited nodes in the network graph
NodeVector	sequential set of nodes selected by an ant
curnode	current node selected by an ant
shipvector	sequential sets of harbor arrival positions selected by an ant per ship or ship route vector
posvector	set of harbor arrival positions selected by an ant
BAntVector	best ship route vector found so far by any ant traversing the graph
cum_time	set of total travel times covered by an ant corresponding to each ship
cum_load	set of total load quantities of external cargoes loaded by each ship corresponding to an ant
Att_a	attraction level of node a from the current node
τ_{ab}	pheromone level of the arc (a,b)
η_{Ra}	heuristic information related to revenue associated with node a
η_{Cab}	heuristic information related to travel cost associated with arc connecting nodes a and b
pr_a	probability assigned to an unvisited node a defining a probability distribution for the whole *NodeSet*
C_{ant}	objective value corresponding to an ant
C_{bs}	objective value of the best ship route vector found so far
M	a very large number
X, Y, Z	sets of values corresponding to binary variables x_{imjnv}, y_{im}, and z_{imv}, derived from final *shipvector* corresponding to an ant

5.1.3 Tour construction. In each iteration we assign a given number of ants at the origin of the network graph. The origin is a dummy node. A single ant traverses the complete graph till it reaches the dummy end node denoted by *Enode*. When an ant traverses through an arc an increase or decrease in pheromone value associated with the arc occurs. This information stored as pheromone values guides following ants in deciding their paths through the graph network. The higher the pheromone contents of an arc, the higher the chances that it will be selected further. The following formulations are used in the ACO algorithm.

When located at node a, an ant moves to another node b chosen according to a pseudorandom proportional rule. The following formulae are use for the same:

$$Att_b = \tau_{ab}^{\alpha}\eta_{Rb}^{\beta_R}/\eta_{Cb}^{\beta_C} \quad \forall b \in NodeSet, a = curnode \tag{37}$$

$$pr_b = Att_b/\left(\sum_{b \in NodeSet} Att_b\right) \quad \forall b \in NodeSet \tag{38}$$

$$b = \begin{cases} \arg\max_{l \in NodeSet}(Att_l) & if\ q \leqslant q_0 \\ J, & otherwise \end{cases} \tag{39}$$

where q is a random variable uniformly distributed in [0,1], q_0 ($0 \leqslant q_0 \leqslant 1$) is a parameter, and J the a random variable selected according to probability distribution given by Equation (38).

5.1.4 Local pheromone update rule. In ACS the ants use a local pheromone update rule that they apply immediately after having crossed an arc (a,b) during the tour construction:

$$\tau_{ab} = (1-\xi)\tau_{ab} + \xi\tau_0 \quad \forall a, b = arc\, just\, traversed \tag{40}$$

where ξ, $0<\xi<1$, and τ_0 are two parameters. The value of τ_0 is set to be the same as initial value of the pheromone trails. The effect of this local updating rule is that each time an ant passes through an arc (i,j), its pheromone trail is reduced, so that the arc becomes less desirable by the following ants. This step avoids stagnation or in other words leads the algorithm away from local optima and toward global exploration.

5.1.5 Global pheromone update rule. In ACS version of ACO algorithm only the best ant so far is allowed to deposit pheromones after each, iteration. The pheromone update is carried out by the following equation. We have added a fixed parameter C_1 in the denominator to make sure that the whole ratio is always positive, since sometimes the C_{bs} may turn out to be negative:

$$\tau_{ab} = (1-\rho)\tau_{ab} + \rho C_0/(C_1 + C_{bs}) \quad \forall Arc(a,b) \in BAntVector \tag{41}$$

5.2 Heuristic description

Figure 3 presents the complete description of the heuristic algorithm. The leftmost part of the flowchart describes the initialization module. As the first step, a corresponding

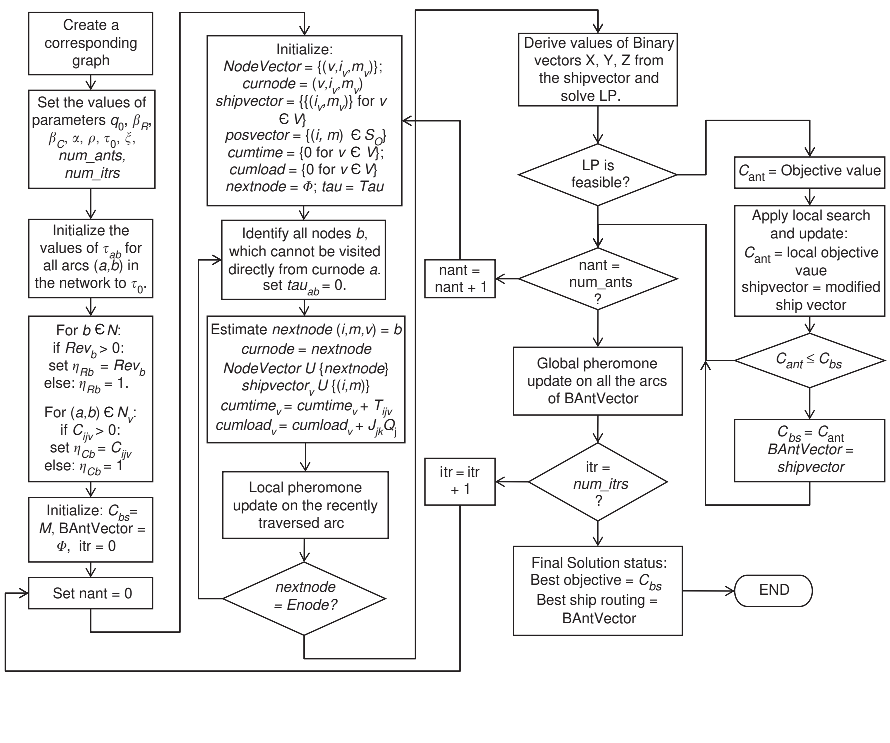

Figure 3.
Detailed flowchart of the algorithm

graph as discussed in Section 5.1.1 is created for the problem based on the given data. Next the values of all the metaheuristic parameters q_0, β_R, β_C, α, ρ, τ_0, ξ, *num_ants* and *num_itrs* needs to be fixed. Next the pheromone value, τ_{ab}, for the feasible arcs connecting the pairs of all nodes a and $b \in N$ are initialized to the value τ_0.

A very important search feature of an ACO algorithm is the use of heuristic information in probabilistic search. We use heuristic information as a ratio of two values in our case. One of the heuristic information is derived from the cost of travel between two nodes. This heuristic information denoted as η_{Cab} and is equated to the average travel cost between two nodes C_{ijv} (where $a = (i,m,v)$ and $b = (j,n,v)$, $\forall v \in V$, $(i,m),(j,n) \in S_N^v$), if they form a feasible path. This heuristic information is taken as a denominator to ensure that higher the travel cost between the current node and a prospective node, lesser are the chances of selection of that node as successive node in the ant route. The second heuristic information is associated with the revenue associated with prospective node b, denoted as η_{Rb}. This is taken as the revenue associated with an optional origin node i, Rev_i. For other nodes this value is taken as equal to unity. This heuristic information value is kept at the nominator to ensure that higher the revenue associated with the prospective arc, the higher the probability of its selection as next node.

In the course of the algorithm the purpose is to derive a combinatorially feasible set of nodes for each ant. The best solution vector found so far by any ant is denoted by *BAntVector* and the objective value associated with the vector is denoted by C_{bs}. The value of C_{bs} is initialized to a very large number denoted by M, as this is a minimization problem.

Next we explain the iterative module of the algorithm. A particular ant in an iteration traverses the graph till a complete route is formed. A variable *curnode* defines the incumbent node of the ant and its initial value is assigned as any one of the nodes corresponding to the starting positions of the ships. A set *shipvector* stores the route formed for each ship. The elements of the subsets for *shipvector* are harbor-arrival nodes, as discussed in the original formulation. The set *shipvector* is initialized as the starting position for each ship. The set *cumtime* stores the cumulative travel time for each ship. In this we only consider the travel time between two successive nodes. The set *cumload* stores the cumulative load onto each ship so far. In this we consider only the external loads, as only they are known in advance. The information contained in the last two sets helps in avoiding the formation of some of the infeasible routes. A set *Tau* stores the updated pheromone values of each arc in the graph. We use a variable set *tau* and initially assign the same values of the set *Tau*. Next we identify the nodes which cannot be directly visited from the incumbent node and assign the value zero to the corresponding entries of the set *tau*. We select the next node for the ant's path using the tour construction steps as discussed in Section 5.1.3. The values of the variables τ_{ab} used in Equation (37) are derived from the set *tau*. This node is assigned to the variable *curnode* and all corresponding entries are updated to the sets *NodeVector*, *shipvector*, *cumtime* and *cumload*, based on the node selected. As soon as an ant traverses an arc a local pheromone update is carried out as per Equation (40). The route building steps are carried out repetitively till the ant automatically chooses the *ENode*. Once a complete ant vector is formed, the next step is to evaluate and update the routing plan.

In the evaluation step we derive the values of binary variables x, y, z from the complete *shipvector* for an ant. The binary values are fixed in the complete MILP formulations for the problem to derive a corresponding LP for a complete routing plan. This LP is solved using a commercial solver (CPLEX) and if it is feasible, the overall objective value for the routing plan is evaluated. This gives the fitness value C_{ant} for the given ant. As an intermediate step we apply a simple local search to check if the current routing plan can be improved by simple adding or removing individual nodes from the current routing plan. If there is an improvement, we update the *shipvector* corresponding to this ant and also the fitness value C_{ant}. If the fitness value derived is lesser than the best solution found so far, the *BAntVector* and C_{bs} are updated with the values of *shipvector* and C_{ant}, respectively.

In each iteration the route building, evaluation/improvement modules are carried out till all the ants are tried. After the end of an iteration the global pheromone update is carried out on all the arcs of the best routing plan, *BAntVector*. The stopping criteria are set twofold; maximum iterations till no improvement or the maximum number of iterations, whichever occurs earlier.

6. Computational results

6.1 Experimental design

The usefulness of solution techniques to a mathematical model can only be tested by extensive data sets of varying sizes and configurations. To the best of our knowledge this extension to the inventory maritime routing problem has not been taken up in academic research. We have not found any published data sets even for the existing variants of inventory maritime routing problem in the extant literature. Al-Khayyal and Hwang (2007) present a framework for the generation of parameters for a multi-commodity maritime inventory routing problem. We modify this framework and add some more parameter sets to generate parameters for the problem. We choose the configuration of our test problems on the basis of the quadruplet ($|V|$, $|H_t|$, $|K_t|$, $|K_{os}|$). Here $|V|$ stands for number of ships. $|Ht|$ represents the number of internal harbors, $|Kt|$ represents the number of internally managed products, and finally $|Kos|$ represents the number of optional external cargoes available for transportation. We take a planning horizon of 10 time units. All the harbors are assumed to be located randomly in a 10×10 Euclidean matrix. The position of any harbor i is denoted by (a_i,b_i). These points are randomly generated by taking $a_i,b_i \sim U[0,10]$, where $U[\alpha,\beta]$ denotes the uniform distribution over the interval $[\alpha,\beta]$. The distance between any two harbors is taken as a Euclidian metric. For simplicity we will take only optional cargoes as external cargoes in our data sets. As explained earlier each optional cargo is associated with one origin and one destination. Appendix presents the framework for generation of parameters for the test problems.

A total of 56 feasible data sets are generated to compare the heuristic performance with that of a MILP solver. All the experiments are carried out on an AMD Athlon™-64, 997 MHz computer with 1.25 GB of RAM running Microsoft Windows XP version 2002. Algorithm is implemented in the programming language Python 2.7.2. We used the academic research edition of CPLEX 12.3 optimization library with Python 2.7.2 as LP/MILP solver. Table I shows all the problem test instances with varying configurations of ships, harbors and cargoes. The last three columns on the number of rows, columns and binaries of reduced MILP are presented to access the real size of the problem instances. On solving an instance on CPLEX, the solver carries out a pre-processing of the input model to reduce the problem size in two to three steps. The solver gives as output the number of rows, columns and binaries in this reduced MILP. It can be clearly seen that two instances of same configuration in terms of number of ships, internal harbors, internal products and optional products may have very different problem sizes, e.g. instances T10 and T11 have very different problem sizes in their reduced MILP.

6.2 Parameter setting for ACO

The first step in applying a modified ACS algorithm to solve the problem of ship routing is to set up the parameters defined in Section 4.1.2. The values of

Instance	No. of ships (|V|)	No. of internal harbors (|Ht|)	No. of internal cargoes (|Kt|)	No. of optional cargoes (|Kos|)	No. of rows in reduced MILP	No. of columns in reduced MILP	No. of binaries in reduced MILP
T1	2	2	1	2	938	228	133
T2	2	2	1	2	941	228	133
T3	2	2	1	2	746	191	104
T4	2	2	1	3	1,497	294	169
T5	2	2	1	3	1,519	300	173
T6	2	2	1	3	1,855	348	211
T7	2	2	1	6	7,703	838	532
T8	2	2	1	6	6,006	689	416
T9	2	2	1	6	6,006	689	416
T10	2	2	2	2	108	59	8
T11	2	2	2	2	1,164	258	127
T12	3	2	2	2	949	242	87
T13	3	2	2	2	1,430	321	159
T14	3	2	2	2	1,420	318	157
T15	3	2	2	3	1,153	282	91
T16	3	2	2	3	2,480	447	231
T17	3	2	2	3	2,792	480	260
T18	3	2	2	4	1,166	267	90
T19	3	2	2	4	3,796	571	302
T20	3	2	2	4	3,676	558	291
T21	3	2	3	3	1,153	282	91
T22	3	2	3	3	2,824	453	226
T23	3	2	3	3	2,443	434	194
T24	3	2	3	6	5,757	707	310
T25	3	2	3	6	9,888	1,019	541
T26	3	2	3	6	9,892	1,024	541
T27	3	3	2	2	1,076	296	109
T28	3	3	2	2	2,470	491	274
T29	3	3	2	2	2,133	446	236
T30	3	3	2	3	3,079	557	276
T31	3	3	2	3	4,830	748	447
T32	3	3	2	3	4,827	746	447
T33	3	3	3	4	3,943	653	280
T34	3	3	3	4	8,090	995	554
T35	3	3	3	4	8,868	1,060	607
T36	3	3	3	6	10,831	1,175	588
T37	3	3	3	6	2,923	500	165
T38	3	3	3	6	12,897	1,275	702
T39	4	3	3	2	1,014	307	86
T40	4	3	3	2	3,719	675	341
T41	4	3	3	2	3,724	679	341
T42	4	3	3	4	2,297	513	163
T43	4	3	3	4	5,632	860	372
T44	4	3	3	4	7,493	994	514
T45	4	4	2	2	2,320	510	252
T46	4	4	2	2	2,121	488	228
T47	4	4	2	2	5,303	942	590
T48	4	4	2	3	5,158	867	443

(*continued*)

Table I.
Test instances for model evaluation

Table I.

Instance	No. of ships (|V|)	No. of internal harbors (|Ht|)	No. of internal cargoes (|Kt|)	No. of optional cargoes (|Kos|)	No. of rows in reduced MILP	No. of columns in reduced MILP	No. of binaries in reduced MILP
T49	4	4	2	3	4,691	789	435
T50	4	4	2	3	5,090	838	469
T51	4	4	2	4	10,737	1,427	857
T52	4	4	2	4	13,325	1,611	1,048
T53	4	4	2	4	13,310	1,616	1,046
T54	5	6	5	6	49,736	3,931	2,207
T55	5	6	5	6	66,865	4,905	2,960
T56	5	6	5	6	62,492	4,661	2,766

parameters ρ, τ_0, ξ, q_0, C_0 and C_1 were fixed at 0.2, 1, 0.1, 0.5, 300 and 200, respectively. We carried out experiments with the values of parameters α, β_R, β_C and *num_ants*. The value for α is considered in the set {0.01, 0.05, 0.1, 0.5, 1, 1.5, 2}, for β_R in the set {0.01, 0.05, 0.1, 0.5, 1, 1.5, 2}, for β_C in the set {0.1, 0.5, 1, 1.5, 2, 2.5, 3} and *num_ants* in the set {15, 20, 25, 28, 35}. Three instances are chosen from the list for experimentations; a small size instance T1, a medium instance T13 and a large instance T24. These instances are solved till 1,000 iterations using all possible combinations of these parameters. The results are sorted in the decreasing order of algorithm performances with respect to best objective value as first priority, time for best solution as second priority and total run time for 1,000 iterations as the last priority. The best combination of these parameters is found to be $\alpha = 2.5$, $\beta R = 0.1$, $\beta C = 0.5$ and *num_ants* = 28.

6.3 Computational experience

We solved all the instances once using MILP solver to obtain the best lower bound on the objective value, MILP objective value and time taken to get optimal solution if possible within the limit. We set the maximum MILP runtime at 3,600 seconds. We use the commercial solver itself to find the best lower bound values on the objective function for larger instances. To get the best lower bound values for each such data sets, we insert the best solution known from our heuristic as starting solution to our MILP, set a CPLEX parameter to guide the solver toward finding best lower bound and run the solver for 7,200 seconds. We solved all the instances three times on the heuristic algorithm to get the average objective value and average time taken to reach best solution. We start by solving the MILP directly for the problem instance up to maximum 3,600 seconds. We note down the results of the MILP solver and solve the problem instance using the heuristic algorithm. Two different stopping criteria are used for the heuristic. If the MILP solves the problem instance to optimality within the run time, we fix the maximum heuristic iterations at 1,000 and stopping criteria as "No improvement in objective for 200 iterations." Table II shows the computational experience with the heuristic algorithm.

As can be seen from Table II, the heuristic performs well as a solution approach. We divide the instances into small, medium and large as per the time taken by MILP solver. The small instances clubbed at the top, are the ones for which the MILP solver reaches optimal solution in a small time. Medium instances grouped in the middle with shaded rows, are the ones for which the solver reaches optimal solution after a long run time. The larger instances shown in bold, are the ones for which the MILP solver could

Instance	LBD	MILP	Heur_avg	T_milp	T_heur_b	T_heur_sc
T1	5.45	5.45	5.45	35.16	6.22	28.04
T2	−0.23	−0.23	−0.23	48.47	3.21	25.85
T3	0.69	0.69	0.69	0.7	3.1	56.3
T4	15.06	15.06	15.06	50.33	73.08	149.45
T10	4.47	4.47	4.47	0.08	0.19	25
T11	15.25	15.25	15.25	2.48	4.1	96.8
T12	3.34	3.34	3.34	1.47	0.25	85.5
T13	−19.33	−19.33	−19.33	211.67	107.92	180.75
T14	−14.01	−14.01	−11.97	64.28	16.2	72
T15	167.98	167.98	167.98	0.52	4.12	61.6
T21	58.35	58.35	58.35	0.99	0.18	60.07
T22	−73.76	−73.76	−73.76	71.23	3.4	197.95
T27	251.92	251.92	251.92	0.6	0.54	123.4
T33	162.17	162.17	162.17	4.33	0.36	204.3
T37	168.89	168.89	168.89	2.11	23.42	225.6
T39	49.18	49.18	49.18	1.09	1.74	101.2
T42	97.15	97.15	97.15	3.34	66.15	298
T45	−3.97	−3.97	−3.97	0.78	0.51	331.67
T46	25.81	25.81	25.81	1.64	41	291.94
T48	74.86	74.86	74.86	10.75	2.08	488.22
T51	126.62	126.62	126.62	181.45	529.43	775.2
T5	−49.41	−49.41	−49.41	925	5.49	59.8
T16	6.52	−26.38	−26.38	1,092.66	29.7	125.99
T17	−10.56	−10.56	−10.46	2,139.14	25.6	207
T20	17.82	17.82	24.78	1,282.27	159	204
T23	−51.88	−51.88	−51.88	766.6	11.5	135.63
T24	235.7	235.7	235.7	1,579.36	26.63	110.96
T30	54.92	54.92	54.92	1,879.6	1.4	309.34
T36	72.01	72.01	72.01	1,659.36	841.7	1,102
T43	59.64	59.64	59.64	2,497	163.4	379.6
T6	**0.235**	**0.235**	**0.235**	**5,752**	**2.91**	**119.12**
T7	**−311.87**	**−36.56**	**−129.93**	**3,600**	**79.8**	**163.95**
T8	**−118.59**	**−50.5**	**−83.11**	**3,600**	**46.12**	**198.02**
T9	**−125.3**	**1.42**	**−83.11**	**3,600**	**120**	**254.9**
T18	**−6.57**	**−14.15**	**−14.1**	**4,318.47**	**5.33**	**196.06**
T19	**−12.045**	**1.9133**	**1.9134**	**3,600**	**11.94**	**162.8**
T25	**−82.88**	**112**	**27.45**	**3,600**	**326.67**	**73.57**
T26	**−147.42**	**−25.42**	**−34.54**	**3,600**	**627.6**	**963.83**
T28	**−151.54**	**−81.54**	**−81.54**	**3,600**	**987.67**	**3,600**
T29	**−47.91**	**−19.77**	**−21.67**	**3,600**	**190.92**	**3,600**
T31	**−250.63**	**−125.05**	**−141.63**	**3,600**	**155.84**	**3,600**
T32	**−91**	**−43.925**	**−44.67**	**3,600**	**493**	**3,600**
T34	**−250.11**	**−42.8**	**−58.36**	**3,600**	**1,792.39**	**3,600**
T35	**−197.893**	**Nil**	**−80.22**	**3,600**	**524.36**	**3,600**
T38	**−210.6949**	**Nil**	**200.18**	**3,600**	**2,413.45**	**3,600**
T40	**−102.9165**	**−42.7412**	**−41.94**	**3,600**	**57.47**	**3,600**
T41	**−132.52**	**−53.53**	**−54.13**	**3,600**	**100.11**	**3,600**
T44	**−312.6923**	**−93.59**	**−124.51**	**3,600**	**45**	**3,600**
T47	**−108.912**	**Nil**	**−126.26**	**3,600**	**1,504.96**	**3,600**
T49	**−202.6427**	**Nil**	**126.64**	**3,600**	**2,117.8**	**3,600**

(*continued*)

Table II.
The values for best lower bound (LBD), MILP solution (MILP), Heuristic average (Heur_avg), MILP time (T_milp), average heuristic time for best solution (T_heur_b), and average heuristic time for stopping criteria (T_heur_sc)

Instance	LBD	MILP	Heur_avg	T_milp	T_heur_b	T_heur_sc
T50	**−112.537**	**−4.6385**	**−6.21**	**3,600**	**2,105.9**	**3,600**
T52	**−328.9813**	**Nil**	**−127.48**	**3,600**	**1,104.9**	**3,600**
T53	**−257.7525**	**Nil**	**−146.91**	**3,600**	**1,949.7**	**3,600**
T54	**17.91**	**Nil**	**98.48**	**3,600**	**2,014.7**	**3,600**
T55	**−449.155**	**Nil**	**65.28**	**3,600**	**103.5**	**3,600**
T56	**−466.7318**	**Nil**	**34.9**	**3,600**	**1,543**	**3,600**

Table II.

Note: The larger instances shown in bold

not find optimal solution within 3,600 seconds. It can be seen that the heuristic provides accurate solutions in comparison to MILP solver for small instances of the problem, but takes somewhat more time in comparison. For medium-sized instances it provides accurate results in much lesser time than MILP solver. In case of large instances the heuristic provides good solutions within a reasonable solution time. Thus the proposed solution methodology seems like a promising approach for complex problems of similar structure to inventory maritime routing.

Thus it can be said that the used solution approach is a viable solution methodology for this type of problem, having a special structure with only binary variables and linear variables in the mathematical formulation. The suggested solution algorithm can be developed further to be used as a practical solution approach, may be as a part of a DSS for large-scale optimization problems of this type.

The suggested solution methodology has scope for improvement. The heuristic can be improved further by incorporating strong local search techniques apart from the one used here. There is also scope of improving the metaheuristic algorithm by incorporating further modification owing to problem structure.

7. Conclusions

Here we address an integrated maritime supply-chain planning problem faced by companies involved in the international outbound logistics of automobiles. We describe the problem under the context of a Roll-on/Roll-off shipping line providing third-party logistics services for finished automobiles to some clients. We present an extension of the multi-commodity inventory maritime routing problem, as discussed in industrial shipping scenarios, to include simultaneous planning of external mandatory and optional cargoes. A MINLP formulation with suitable linearization scheme is presented for the problem.

It is being proved in the literature that this type of problem is very complex to solve reasonable size problems using standard MIP solvers. We propose a novel heuristic approach based on the combination of a type of ACO metaheuristic and linear programming. We tested the model using problem instances of varying sizes. The solution method performs well on all the test cases and could be used to solve real-life problem instances. The heuristic can be improved further by incorporating strong local search techniques apart from the one used here. There is also scope of improving the metaheuristic algorithm by incorporating further modification owing to problem structure. The research problem explained above can be further extended by incorporating variable demand and supply with time, deciding the production schedule along with shipping plan, etc.

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Further reading

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Appendix

Parameter	Meaning	Distribution (value)
N_{os}	Number of optional harbors	$\lvert K_{os}\rvert \times 2$, where K_{os} is the set of optional cargoes
N_{os0}	First optional harbor	$\lvert H_t\rvert + 1$, where H_t is the set of internal harbors
T	Time horizon	10
TW_{si}	Starting time window for external harbor i	$U\,[0,T/5]$ for $i\in[N_{os0}, N_{os0}+Nos/2]$ $U\,[3T/4,T]$ for $i\in[N_{os0}, N_{os0}+N_{os}/2+1, N_{os0}+N_{os}]$
TW_{ei}	Ending time window for external harbor i	$TW_{si}+U\,[0,T/3]$ for $i\in[N_{os0}, N_{os0}+Nos/2]$ $\mathrm{Min}(TW_{si}+\mathrm{U}[0,T/3],T)$ for $i\in[N_{os0}+N_{os}/2+1, N_{os0}+N_{os}]$

Table AI.
Parameter generation for test problems

(*continued*)

Parameter	Meaning	Distribution (value)
J_{ik}	$+1$, if port i produces product k, -1 otherwise	We assign $-1/+1$ values with probability of 0.5 for internal cargoes/harbors and for external harbors we assign $+1$ value to origin port and -1 to destination port corresponding to an optional cargo
R_{ik}	Production/consumption rate of product k at port i	$U[1,6]$
w_v	Proportionality constant for estimation of travel cost	$U[0.5,1]$
CAP_v	Capacity of ship v	$\vert K\vert \times U[20,70]$
Q_{vk}	Initial load of cargo k on ship v	$U[0,CAPv/\vert K\vert]$
$S_{\max_i}$	Maximum allowable stock capacity at port i	$\vert K_t\vert \times U[20,70]$
$S_{\min_i}$	Minimum allowable stock capacity at port i	0
Cw_{ik}	Cost of loading/discharging a single unit of product k at port i	$U[0.0005,0.001]$
IS_{ik}	Initial inventory of product k a harbor i	$(\text{Smax}_i/\vert K_t\vert) \times U[0.3,0.7]$
TQ_{ik}	Time required for loading/discharging a single unit of product k at port i	$U[0,0.03]$
C_{ijv}	Travel cost for ship v from port i to port j	$w_v\sqrt{(a_i-a_j)^2+(b_i-b_j)^2}$,where (a_i,b_i) and (a_j,b_j) are Euclidean coordinates of ports i and j respectively. Here $a_i,a_j,b_i,b_j \sim U[0.10]$
T_{ijv}	Travel time for ship v from port i to port j	$T/8+0.04\sqrt{(a_i-a_j)^2+(b_i-b_j)^2}$,where (a_i,b_i) and (a_j,b_j) are Euclidean coordinates of ports i and j respectively. Here $a_i,a_j,b_i,b_j \sim U[0.10]$
$RevI_{ik}$	Revenue per unit of internal product k when delivered at harbor i	$C_{ijv\max} \times U[0,10]$, where $C_{ijv\max} = \max_{i,j,v}(C_{ijv}/CAP_v)$
RT_v	Benefit per unit time associated with finishing a ship v's route early.	$\max_{i,j,v}(C_{ijv}/T_{ijv})\times U[0,5]$
CI_{ik}	Per unit per day inventory holding cost of product k at harbor i	$\max_{i,j,v}(C_{ijv}/CAP_v)\times 0.1\times U[0.4]$
$Q_i(=Q_{i+Nos/2})$	Quantity to be serviced at an optional external harbor i	$\max_{v}(CAP_v)\times U[0,1]$
Rev_i	Revenue associated with an optional port i	$\max_{v}(C_{i(i+N_{os}2)v}/CAP_v)\times Q_i\times 10\times U[0,1]$

Table AI.

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